EUROPEAN PATENT SPECIFICATION

(45) Date of publication and mention of the grant of the patent:
07.04.2010 Bulletin 2010/14

(21) Application number: 02724362.5

(22) Date of filing: 28.05.2002

(51) Int Cl.:
G06K 9/36 (2006.01) H04N 7/26 (2006.01)
G06T 9/00 (2006.01)

(54) ARCHITECTURES FOR DISCRETE WAVELET TRANSFORMS
ARCHITEKTUREN FÜR DISKRETE WAVELET-TRANSFORMATIONEN
ARCHITECTURES POUR TRANSFORMEES EN ONDELETTES DISCRETES

(84) Designated Contracting States:
AT BE CH CY DE DK ES FI FR GB GR IE IT LI LU MC NL PT SE TR

(30) Priority: 01.06.2001 US 872682

(43) Date of publication of application:

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• OMAKI R.Y. ET AL.: 'VLSI implementation of a realtime wavelet video coder'CUSTOM INTEGRATED CIRCUITS CONFERENCE, IEEE 2000, pages 543 - 546, XP002955173

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Description

BACKGROUND OF THE INVENTION

[0001] The invention relates to architectures for implementing discrete wavelet transforms (DWTs). The invention relates to any field where DWTs may be in use which is particularly but not exclusively concerned with architectures used in the fields of digital signal and image processing, data compression, multimedia, and communications.

[0002] A list of documents is given at the end of this description. These documents are referred to in the following by their corresponding numeral in square brackets.

[0003] The Discrete Wavelet Transform (DWT) [1]-[4] is a mathematical technique that decomposes an input signal of length $N = r \times k^m$ in the time domain by using dilated/contracted and translated versions of a single basis function, named the prototype wavelet. In one particular case $N=2^m$ (i.e. $r = 1$ and $k = 2$). DWTs can be performed using Haar wavelets, Hadamard wavelets and wavelet packets. Decomposition by Haar wavelets involves low-pass and high-pass filtering followed by down-sampling by two of both resultant bands and repeated decomposition of the low-frequency band to $J$ levels or octaves.

[0004] In the last decade, the DWT has often been found preferable to other traditional signal processing techniques since it offers useful features such as inherent scalability, computational complexity of $O(N)$ (where $N$ is the length of the processed sequence), low aliasing distortion for signal processing applications, and adaptive time-frequency windows. Hence, the DWT has been studied and applied to a wide range of applications including numerical analysis [5]-[6], biomedicine [7], image and video processing [1], [8]-[9], signal processing techniques [10] and speech compression/decompression [11]. DWT based compression methods have become the basis of such international standards as JPEG 2000 and MPEG-4.

[0005] In many of these applications, real-time processing is required in order to achieve useful results. Even though DWTs possess linear complexity, many applications cannot be handled by software solutions only. DWT implementations using digital signal processors (DSPs) improve computation speeds significantly, and are sufficient for some applications. However, in many applications software DWT implementations on general purpose processors or even DSP implementations are too slow. Therefore, the implementation of the DWT by means of dedicated very large scale integrated (VLSI) Application Specific Integrated Circuits (ASICs) has recently captivated the attention of a number of researchers, and a number of DWT architectures have been proposed [12]-[24]. Some of these devices have been targeted to have a low hardware complexity. However, they require at least $2N$ clock cycles (cc's) to compute the DWT of a sequence having $N$ samples. Nevertheless, devices have been designed having a period of approximately $N$ cc's (e.g., the three architectures in [14] when they are provided with a doubled hardware, the architecture A1 in [15], the architectures in [16]-[18], the parallel filter in [19], etc.). Most of these architectures use the Recursive Pyramid Algorithm (RPA) [26], or similar scheduling techniques, in order both to reduce memory requirement and to employ only one or two filter units, independently of the number of decomposition levels (octaves) to be computed. This is done by producing each output at the "earliest" instance that it can be produced [26].

[0006] The architectures presented in [17] and [18] consist of two pipeline stages where the first pipeline stage implements the first DWT octave and the second stage implements all of the following octaves based on the RPA. Even though the architectures of [17] and [18] operate at approximately 100% hardware utilisation for a large enough number of DWT octaves, they have complex control and/or memory requirements. Furthermore, because they employ only two pipelining stages they have relatively low speeds. The highest throughput achieved in conventional architectures is $N = 2^m$ clock cycles for implementing a $2^m$-point DWT. Approximately 100% hardware utilisation and higher throughput is achieved by the architectures previously proposed in [31], [32] and [34]. The FPP and LPP architectures presented in [34] are dependent on the length of the DWT filters and, for the case of the FPP architecture, on the length of the input signal.

[0007] The demand for low power VLSI circuits in modern mobile/visual communication systems is increasing. Improvements in VLSI technology have considerably reduced the cost of the hardware. Therefore, it is often worthwhile reducing the period, even at the cost of increasing the amount of hardware. One reason is that low-period devices consume less power. For instance, a device D having a period $T=N/2$ cc's can be employed to perform processing which is twice as fast as a device D' having a period $T=N$ cc's. Alternatively, if the device D is clocked at a frequency $f$ then it can achieve the same performance as the device D' clocked at a frequency $f'=2f$. Therefore, for the device D the supply voltage (linear with respect to $f$) and the power dissipation (linear with respect to $f^2$) can be reduced by factors of 2 and 4 respectively with respect to the supply voltage of the device D' [27].

[0008] High throughput architectures typically make use of pipelining or parallelism in which the DWT octaves are implemented with a pipeline consisting of similar hardware units (pipeline stages). Even though pipelining has been already exploited by existing DWT architectures (e.g., those in [12], [23]-[24]), the fastest pipelined designs need at least $N$ time units to implement an $N$-point DWT.

[0009] Most of the known designs for implementation of DWTs are based on the tree-structured filter bank represen-
tation of DWT shown in Figure 1 where there are several \((J)\) stages (or octaves) of signal decomposition each followed by down-sampling by a factor of two. As a consequence of downsampling, the amount of data input to each subsequent decomposition stage is half the amount input to the immediately previous decomposition stage. This makes the hardware of decomposition stages in a typical pipelined device designed to implement DWT using the tree-structured approach heavily under-utilised, since the stage implementing the octave \(j = 1,...,J\) is usually clocked at a frequency \(2^{-(j-1)}\) times lower than the clock frequency used in the first octave [24]. This under-utilisation comes from a poor balancing of the pipeline stages when they implement the DWT octaves and leads to low efficiency.

[0010] In [30] a pipeline architecture has been proposed based on the tree-structured filter bank representation which achieves approximately 100% hardware utilisation and throughput of \(N/2 = 2^{m-1}\) clock cycles for a \(2^m\)-point DWT. This involves a \(J\)-stage pipeline using, as far as it is possible, half as many processing units from one stage to the next stage. In [34]-[35], a flowgraph representation of DWTs was proposed and compared to the tree-structured filter bank representation to produce parallel/pipelined DWT architectures. In particular, the flowgraph representation fully reveals the parallelism inherent in every octave, as well as demonstrating the data transfers within and between octaves. This allows pipelining and parallelism to be combined to achieve higher throughput and hardware utilisation. In particular, the flowgraph representation demonstrates that the \(j^{\text{th}}\) octave, \(j = 1,...,J\), of a \(2^m\)-point DWT needs \(2^m/j\) independent similar operations which can be implemented fully or partially in parallel. DWT octaves are implemented in a pipelined mode while every octave is implemented in a parallel mode and the level of parallelism is halved from one octave to the next. Incorporating a varying level of parallelism within pipeline stages enables parallel-pipelined devices with perfectly balanced pipeline stages to be designed. This means implementing octaves in a pipelined mode where the pipeline stages are parallelized to varying degrees from stage to stage. This idea can be realized in numerous different ways. In [34], two architectures referred to as fully parallel-pipelined (FPP) and limited parallel-pipelined (LPP) DWT architectures were proposed. Both architectures consist of \(J\) pipeline stages, each pipeline stage containing half the number of processor elements compared to the previous stage. As a result very high throughput and approximately 100% hardware utilisation is achieved.

[0011] Known parallel or pipelined architectures essentially depend on DWT parameters such as input length, the number of octaves, the length and, in some cases, the actual coefficient values of the low-pass and high-pass filters. For larger values of these parameters, these architectures can be very large. Furthermore, it is only possible to implement a DWT with fixed parameters within a given hardware realization of a given architecture. However, in JPEG 2000, a DWT is separately applied to tiles of an image, in which the sizes of the tiles may vary from 1 to \(2^{32} - 1\).

[0012] The number of octaves of decomposition may vary from 0 to 255 for different tiles. Thus, it is desirable to have a device capable of implementing DWTs with varying parameters or, in other words, a unified device that is relatively independent of the DWT parameters. Designing such a device is straightforward in the case of serial architectures. It is not so straightforward in the case of parallel or pipelined architectures.

[0013] Most of the conventional architectures [12]-[26] employ a number of multipliers and adders proportional to the length of the DWT filters. Even though some architectures [17], [18] are capable of implementing DWTs with varying number of octaves, their efficiency decreases rapidly as the number of octaves increases.

SUMMARY OF THE INVENTION

[0014] According to the aspects of the invention which are set out in the appended claims, the present invention is directed to a microprocessor structure for performing a discrete wavelet transform operation. In one embodiment, the discrete wavelet transform operation comprises decomposition of an input signal vector comprising a number of input samples, over a specified number of decomposition levels \(j\), where \(j\) is an integer in the range 1 to \(J\), starting from a first decomposition level and progressing to a final decomposition level. The microprocessor structure has a number of processing stages, each of the stages corresponding to a decomposition level \(j\) of the discrete wavelet transform and being implemented by a number of basic processing elements. The number of basic processing elements implemented in each of the processing stages decreases by a constant factor at each increasing decomposition level \(j\).

[0015] The microprocessor structure is a generally scalable structure based on the flowgraph representation of the Discrete Wavelet Transform.

[0016] In this invention, general parametric structures of two types of DWT architectures, referred to as Type 1 and Type 2 core DWT architectures are introduced, as well as general parametric structures of two other DWT architectures which are constructed based on either a core DWT architecture and are referred to as the multi-core DWT architecture and the variable resolution DWT architecture, respectively. All the architectures can be implemented with a varying level of parallelism thus providing an opportunity to determine the amount of hardware resources required in a particular application and to make a trade-off between computational speed, cost, chip area and power consumption requirements. In this invention advantages of both parallel and pipelined processing are combined in order to develop DWT architectures with improved efficiency (hardware utilisation) and, consequently, with improved throughput or power consumption. General structures of several DWT architectures operating at approximately 100% hardware utilisation at every level of
parallelism are proposed. The architectures presented are relatively independent of the size of the input signal, the
length of the DWT filters, and in the case of a variable resolution DWT architecture, also on the number of octaves and
can be implemented with varying levels of parallelism. In addition, the architectures demonstrate excellent area-time
characteristics compared to the existing DWT designs. The invention provides architectures which are regular and easily
controlled, which do not contain feedback, long (depending on the length of the input) connections or switches. They
can be implemented as semisystolic arrays.

BRIEF DESCRIPTION OF THE DRAWINGS

[0017] Embodiments of the invention will now be described, by way of example only, with reference to the accompanying
drawings in which:

Figure 1 shows the tree-structured definition/representation of DWTs on which most of the known DWT architectures
are based;
Figure 2 shows an example of a new flowgraph representation of DWTs on which the architectures proposed in this
invention are based;
Figure 3 shows an embodiment of a compact form flowgraph representation of DWTs;
Figure 4 shows the general architecture of two types of core DWT architectures according to the invention;
Figure 5 shows a possible embodiment of one stage of the Type 1 architecture of Figure 4;
Figure 6 shows an embodiment of the Type 1 core DWT architecture embodiment of Figure 5 corresponding to
the parameters: p=Lmax=6; J=3; N=2^m, m=3, 4,...;
Figure 7 shows another embodiment of the Type 1 Core DWT architecture embodiment of Figure 5 corresponding
to the parameters: p=Lmax=6; J=3; N=2^m, m=3, 4,...;
Figure 8 shows four possible embodiments of processing elements (PEs) that can be used in the Type 1 core DWT
architecture of Figure 4;
Figure 9 shows a possible embodiment of one stage of the Type 2 core DWT architecture of Figure 4;
Figure 10 shows an embodiment of the possible realisation of the Type 2 core DWT architecture;
Figure 11 shows the general architecture of the multi-core DWT architecture according to the invention;
Figure 12 shows an embodiment of the possible realisation of the Type 2 core DWT architecture;

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

[0018] To describe architectures proposed in this invention we first need to define the DWT and to present the basic
algorithm that is implemented within the architectures. There are several alternative definitions/representations of DWTs
such as the tree-structured filter bank, the lattice structure, lifting scheme or matrix representation. The following dis-
cussion uses the matrix definition and a flowgraph representation of DWTs, which is very effective in designing efficient
parallel/pipelined DWT architectures.

[0019] A discrete wavelet transform is a linear transform \( y = H \cdot x \), where \( x = [x_0,...,x_{N-1}]^T \) and \( y = [y_0,...,y_{N-1}]^T \) are the
input and the output vectors of length \( N = 2^m \), respectively, and \( H \) is the DWT matrix of order \( N \times N \) which is formed as
the product of sparse matrices:

\[
H = H^{(J)} H^{(J-1)} \ldots H^{(1)}, \quad 1 \leq J \leq m; \\
H^{(j)} = \begin{pmatrix}
D_j & 0 \\
0 & I_{2^m-2^{m-j+1}}
\end{pmatrix}, \quad j = 1,\ldots,m
\]  \hspace{1cm} (1)

where \( I_k \) is the identity \( (k \times k) \) matrix \( (k = 2^m - 2^{m-j+1}) \), and \( D_j \) is the analysis \( (2^{m-j+1} \times 2^{m-j+1}) \) matrix at stage \( j \) having
the following structure:
where \( LP = [l_1, \ldots, l_L] \) and \( HP = [h_1, \ldots, h_L] \) are the vectors of coefficients of the low-pass and of the high-pass filters, respectively (\( L \) being the length of the filters), and \( P_j \) is the matrix of the perfect unshuffle operator of the size \((2^{m-j+1} \times 2^{m-j+1})\). The perfect unshuffle operator collects the even (odd) numbered components of an input vector at the first (second) half of the output vector assuming the component numbering begins from zero. For the sake of clarity both the low-pass and the high-pass filters are assumed to have the same length, which is an even number. The result may be readily expanded to the general case of arbitrary filter lengths. In the general case (that is where \( N = r \times 2^m \)), where \( k \) is not equal to two (that is there are other than two filtering operations), a suitable stride permutation rather than the unshuffle operation is applied.

Adopting the representation introduced in equations (1) and (2), the DWT is computed in \( J \) stages (also called decomposition levels or octaves), where the \( j \) th stage, \( j = 1, \ldots, J \), constitutes multiplication of a sparse matrix \( H^{(j)} \) by a current vector of scratch variables, the first such vector being the input vector \( x \). Noting that lower right corner of every matrix \( H^{(j)} \) is an identity matrix and taking into account the structure of the matrix \( D_j \), the corresponding algorithm can be written as the following pseudocode where \( x_{LP}^{(j)} = \left[ (x_{LP}^{(0)}(0), \ldots, x_{LP}^{(2^m-j-1)}) \right]^T \), and

\[
x_{HP}^{(j)} = \left[ (x_{HP}^{(0)}, \ldots, x_{HP}^{(2^m-j-1)}) \right]^T, \quad j = 1, \ldots, J,
\]

are \((2^{m-j} \times 1)\) vectors of scratch variables, and the notation \([x_1^T, \ldots, x_k^T]^T\) stands for concatenation of column vectors \( x_1, \ldots, x_k \).
Algorithm 1.

1. Set $x^{(0)}_{LP} = [x^{(0)}_{LP}(0), \ldots, x^{(0)}_{LP}(2^m-1)]^T = x^{(0)}$

2. For $j = 1, \ldots, J$ compute

   $$x^{(0)}_{LP} = [x^{(0)}_{LP}(0), \ldots, x^{(0)}_{LP}(2^{m-j}-1)]^T \quad \text{and} \quad x^{(0)}_{HP} = [x^{(0)}_{HP}(0), \ldots, x^{(0)}_{HP}(2^{m-j}-1)]^T$$

   where

   $$[(x^{(0)}_{LP})^T, (x^{(0)}_{HP})^T]^T = D_j \cdot x^{(j-1)}_{LP}, \quad (3)$$

   or, equivalently,

2. For $i = 0, \ldots, 2^{m-j} - 1$,

   Begin

   Form the vector // a subvector of length $L$ of the vector $x^{(j-1)}_{LP} \star//$

   $$\tilde{x} = [x^{(j-1)}_{LP}(2i), x^{(j-1)}_{LP}(2i+1), x^{(j-1)}_{LP}((2i+2) \mod 2^{m-j+1}), \ldots, x^{(j-1)}_{LP}((2i + L - 1) \mod 2^{m-j+1})]^T$$

   Compute

   $$x^{(j)}_{LP}(i) = L \cdot \tilde{x} ; \quad x^{(j)}_{HP}(i) = H \cdot \tilde{x} ;$$

   End

3. Form the output vector

   $$y = [x^{(j)}_{LP}, x^{(j)}_{HP}, x^{(j-1)}_{HP}, \ldots, x^{(2)}_{HP}, x^{(1)}_{HP}]^T.$$

[0021] Computation of Algorithm 1 with the matrices $D_j$ of equation (2) can be demonstrated using a flowgraph representation. An example for the case $N = 2^3 = 8$, $L = 4$, $J = 3$ is shown in Figure 2. The flowgraph consists of $J$ stages, the $j$-th stage, $j = 1, \ldots, J$, having $2^{m-j}$ nodes (depicted as boxes on Figure 2). Each node represents a basic DWT operation (see Figure 2(b)). The $j$-th node, $i = 0, \ldots, 2^{m-j} - 1$, of stage $j=1,\ldots,J$ has incoming edges from $L$ circularly consecutive nodes $2i, 2i+1, (2i+2) \mod 2^{m-j+1}, \ldots, (2i + L - 1) \mod 2^{m-j+1}$ of the preceding stage or (for the nodes of the first stage) from inputs. Every node has two outgoing edges. An upper (lower) outgoing edge represents the value of the inner product of the vector of low-pass (high-pass) filter coefficients with the vector of the values of incoming edges. Outgoing values of a stage are permuted according to the perfect unshuffle operator so that all the low-pass components (the values of upper outgoing edges) are collected in the first half and the high-pass components are collected in the second half of the permuted vector. Low pass components then form the input to the following stage or (for the nodes of the last stage) represent output values. High-pass components and the low pass components at that stage represent output values at a given resolution.

[0022] Essentially, the flowgraph representation provides an alternative definition of discrete wavelet transforms. It
has several advantages, at least from the implementation point of view, as compared to the conventional DWT representations such as the tree-structured filter bank, lifting scheme or lattice structure representation.

[0023] However, the flowgraph representation of DWTs as it has been presented has a disadvantage of being very large for bigger values of \( N \). This disadvantage can be overcome based on the following. Assuming \( J < \log_2 N \) i.e. the number of decomposition levels is significantly smaller than the number of points in the input vector (in most applications \( J < \log_2 N \) ) one can see that the DWT flowgraph consists of \( N/2^J \) similar patterns (see the two hatched regions on Figure 2). Each pattern can be considered as a 2\(^J\)-point DWT with a specific strategy of forming the input signals to each of its octaves. The \( 2^{m+1} \) input values of the \( J \) th, \( j = 1, \ldots, J \), octave are divided within the original DWT (of length \( N = 2^m \) ) into \( N/2^J = 2^{m-J} \) non-overlapping groups consisting of \( 2^{J-j+1} \) consecutive values. This is equivalent to dividing the vector \( x^{(s)} \) of (3) into subvectors \( x^{(s)} = x^{(s)}_L \cdot (s \cdot 2^{J-j-1} + (s + 1) \cdot 2^{J-j+1} - 1), \) \( s = 0, \ldots, 2^{m-J} - 1 \), where here and in the following the notation \( x(a : b) \) stands for the subvector of \( x \) consisting of the \( a \)-th to \( b \)-th components of \( x \). Then the input of the \( j \)-th, \( j = 1, \ldots, J \), octave within the \( s \)-th pattern is the subvector \( x^{(s)}_L (0:2^{J-j+1}+L-3) \) of the vector,

\[
\hat{x}^{(1-L)}_{[s, J]} = \left[ \begin{array}{c} x^{(1)}_{L, \cdot (s, J)} \\ (s + 1) \cdot 2^{J-j+1} - 1) \\ \ldots \\ \begin{pmatrix} x^{(J-j)}_{L, \cdot (s, J)} - s + (s + 1) \cdot 2^{J-j-1} \\ (s + 1) \cdot 2^{J-j+1} - 1) \\ \ldots \\ x^{(J)}_{L, \cdot (s, J)} - s + (s + 1) \cdot 2^{J-j-1} \\ (s + 1) \cdot 2^{J-j+1} - 1) \end{pmatrix} 
\end{array} \right]^T
\]

being the concatenation of the vector \( x^{(1-L)}_{[s, J]} \) with the circularly next \( Q_j \) vectors where \( Q_j = \lceil (L-2)/2^{J-j+1} \rceil \).

[0024] If the \( 2^{m-J} \) patterns are merged into a single pattern, a compact (or core) flowgraph representation of the DWT is obtained. An example of a DWT compact flowgraph representation for the case \( J=3, L=4 \) is shown in Figure 3. The compact DWT flowgraph has \( 2^{J-j} \) nodes at its \( j \)-th, stage, \( j = 1, \ldots, J \), where a set of \( 2^{m-J} \) temporally distributed values are now assigned to every node. Every node has \( L \) incoming and two outgoing edges like in the ("non-compact") DWT flowgraph. Again incoming edges are from \( L \) "circularly consecutive" nodes of the previous stage but now every node represents a set of temporally distributed values. Namely, the \( L \) inputs of the \( j \)-th node of the \( j \)-th, stage, \( j = 1, \ldots, J \), for its \( s \)-th value, \( s = 0, \ldots, 2^{m-J} - 1 \) are connected to the nodes \((2i+n)2^{J-j+1}, n = 0, \ldots, L-1 \) of the \((j-1)\)st stage which now represent their \( (s+s') \)th values where \( s' = (2i+n)2^{J-j+1} \). Also, outputs are now distributed over the outgoing edges of the compact flowgraph not only spatially but also temporally. That is, each outgoing edge corresponding to a high-pass filtering result of a node or low-pass filtering result of a node of the last stage represents a set of \( 2^{m-J} \) output values. Note that the structure of the compact DWT flowgraph does not depend on the length of the DWT but only on the number of decomposition levels and filter length. The DWT length is reflected only in the number of values represented by every node. It should also be noted that the compact flowgraph has the structure of a \( 2^J \)-point DWT with slightly modified appending strategy. In fact, this appending strategy is often used in matrix formulation of the DWT definition.

[0025] Let \( \hat{D}_j \) denote the main \((2^{J-j+1} \times (2^{J-j+1}+L-2))\)-minor of \( D_j \) (see equation (2)), \( j = 1, \ldots, J \), that is let \( \hat{D}_j \) be a matrix consisting of the first \( 2^{J-j+1} \) rows and the first \( 2^{J-j+1}+L-2 \) columns of \( D_j \). For example, if \( J+j+1=2 \) and \( L = 6 \), then \( \hat{D}_j \) is of the form:

\[
\hat{D}_j = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 0 & 0 \\
0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 2 & 3 & 4 & 5 & 6 & 0 & 0 \\
0 & 0 & 1 & 2 & 3 & 4 & 5 & 6
\end{pmatrix}
\]

[0026] Adopting the notation of equation (4), the computational process represented by the compact flowgraph can be described with the following pseudocode.
Algorithm 2.

1. For $s = 0, \ldots, 2^{m-J} - 1$ set $x_{LP}^{[0,j]} = x(s \cdot 2^j : (s+1) \cdot 2^j - 1)$;

2. For $j = 1, \ldots, J$

   For $s = 0, \ldots, 2^{m-J} - 1$

   Begin
   
   2.1. Set $\hat{x}^{[0,j]}$ according to equation (4)
   
   2.2. Compute $\left[ \left( x_{LP}^{[0,j]} \right)^T, \left( x_{HP}^{[0,j]} \right)^T \right]^T = \hat{D}_j \cdot \hat{x}^{[0,j]}[0:2^{j-1}+L-3]$

   End

3. Form the output vector

   $y = \left[ \left( x_{LP}^{[1,0]} \right)^T, \ldots, \left( x_{LP}^{[1,J-1]} \right)^T, \left( x_{LP}^{[2,0]} \right)^T, \ldots, \left( x_{LP}^{[2,J-1]} \right)^T, \ldots, \left( x_{HP}^{[1,0]} \right)^T, \ldots, \left( x_{HP}^{[1,J-1]} \right)^T, \ldots, \left( x_{HP}^{[2,0]} \right)^T, \ldots, \left( x_{HP}^{[2,J-1]} \right)^T \right]^T$.

[0027] Implementing the cycle for $s$ in parallel yields a parallel DWT realisation. On the other hand, by exchanging the nesting order of the cycles fou $j$ and $s$ and implementing the (nested) cycle fou $j$ in parallel it is possible to implement a pipelined DWT realisation. However, both of these realisations would be inefficient since the number of operations is halved from one octave to the next. However, combining the two methods yields very efficient parallel-pipelined or partially parallel-pipelined realisations.

[0028] To apply pipelining to the Algorithm 2, retiming must be applied since computations for $s$ include results of computations for $s+1, \ldots, s+Q_j-1$ meaning that the $j$th octave, $j = 1, \ldots, J$, introduces a delay of $Q_j$ steps. Since the delays are accumulated, computations for the $j$th octave, $j = 1, \ldots, J$, must start with a delay of

$$s^*(j) = \sum_{n=1}^{j} Q_n.$$  

(5)

during the steps $s = s^*(j), \ldots, 2^{m-J} + s^*(j) - 1$. Thus, computations take place starting from the step $s = s^*(1)$ until the step $s = s^*(J) + 2^{m-J} - 1$. At steps $s = s^*(1), \ldots, s^*(2)-1$ computations of only the first octave are implemented, at steps $s = s^*(2), \ldots, s^*(3)-1$ only operations of the first two octaves are implemented, etc. Starting from step $s = s^*(J)$ until the step $s = s^*(1) + 2^{m-J} - 1$ (provided $s^*(J) < s^*(1) + 2^{m-J}$) computations of all the octaves $j = 1, \ldots, J$ are implemented, but starting from step $s = s^*(1) + 2^{m-J}$ no computations for the first octave are implemented, starting from step $s = s^*(2) + 2^{m-J}$ no computations for the first two octaves are implemented, etc. In general, at step $s = s^*(1) \ldots 2^{m-J} + s^*(J)-1$ computations for octaves $j = J_1, \ldots, J_2$ are implemented where $J_1 = \min\{j\}$ such that $s^*(j) \leq s \leq s^*(j) + 2^{m-J}$ and $J_1 = \max\{j\}$ such that $s^*(j) \leq s < s^*(j) + 2^{m-J}$. The following pseudocode presents the pipelined DWT realisation which is implemented within the architectures proposed in this invention.
Algorithm 3.

1. For $s = 0, \ldots, 2^m - 1$ set $x^{(s,J)}_{LP} = x(s \cdot 2^J : (s + 1) \cdot 2^J - 1)$;

2. For $s = s^*(1), \ldots, 2^m - 1$ do in parallel

   Begin

   2.1. Set $\hat{x}^{(L-s^*(0))}$ according to (4)

   2.2. Compute

   \[
   \left[ (x^{(L-s^*(0))}_{LP})^T, (x^{(L-s^*(0))}_{HP})^T \right]^T = \tilde{D}_J \cdot \hat{x}^{(L-s^*(0))}(0; 2^{J-1} + L - 3).
   \]

   End

3. Form the output vector

\[
y = \left[ (x^{(0,0)}_{LP})^T, (x^{(1,0)}_{LP})^T, \ldots, (x^{(L-1,0)}_{LP})^T \right]^T.
\]
multiplications and additions are obtained in parallel. Thus the time period (measured as the intervals between time units when successive input vectors enter to the architecture) is equal to $2^{m-J[\log_2 L]}$ time units where the duration of the time unit is equal to the period of one multiplication-operation. This is $2^{J/j \log_2 L} / p$ time units faster than the best period of previously known architectures [12-26] and $2^{J/J' \log_2 L} / p$ time units faster than the architectures described in [30]. The efficiency (or hardware utilisation) of both architectures is equal to $L(j/j' \log_2 L) \times 100\%-100\%$. In the case of $p = L = L_{\text{max}}$, the period is $2^{m-J}$ time units which is the same as for the FPP architecture presented in [34] and [35], which, however, depends on the filter length $L$ (i.e. the LPP architecture is only able to implement DWTs with filters of a fixed length $L$). The two types of core DWT architecture differ according to the absence (Type 1) or presence (Type 2) of interconnection between the PEs of one pipeline stage. Possible realisations of the two types of core DWT architectures are presented in Figures 5 to 10. The two types of core DWT architecture described above may be implemented with a varying degree of parallelism depending on the parameter $p$.

Further flexibility in the level of parallelism is achieved within multi-core DWT architectures by introducing a new parameter $r = 1, ..., 2^{m-J}$. The multi-core DWT architecture is, in fact, obtained from corresponding (single-)core DWT architecture by expanding it times. Its general structure is presented in Figure 11. The architecture consists of a serial or parallel data input block and $J$ pipeline stages, the $j$th pipeline stage, $j = 1, ..., J$, consisting of a data routing block and $r2^{J/j}\!$ PEs. The time period of the multi-core DWT architecture is equal to $(2^{m-J[\log_2 L]}) / r$ time units which is $r$ times faster than that of single-core DWT architecture, i.e. a linear speed-up is provided. The efficiency of the multi-core DWT architecture is the same as for single-core architectures, that is, approximately $100\%$. Note that in the case of $p = L = L_{\text{max}}$ and $r = 2^{m-J}$ the period is just one time unit for a $2^m$-point DWT. A similar performance is achieved in the FPP architecture presented in [34] and [35], which can be considered as a special case ($p = L = L_{\text{max}}$ and $r = 2^{m-J}$) of a possible realisation of the multi-core DWT architecture.

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The proposed DWT Architectures

This section presents the general structures of the two types of DWT architecture, referred to as Type 1 and Type 2 core DWT architectures, as well as two other DWT architectures which are constructed based on either of the core DWT architectures and are referred to as the multi-core DWT architecture and the variable resolution DWT architecture, respectively. The multi-core DWT architecture is an extension of either one of the core DWT architectures, which can be implemented with a varying level of parallelism depending on a parameter $r$, and in a particular case ($r = 1$) it becomes the single-core DWT architecture. For ease of understanding, the presentation of the architectures starts with a description of the single-core DWT architectures.

Both types of core DWT architecture implement an arbitrary discrete wavelet transform with $J$ decomposition levels (octaves) based on low-pass and high-pass filters having a length $L$ not exceeding a given number $L_{\text{max}}$. Their operation is based on Algorithm 3 presented earlier. The general structure representing both types of core DWT architecture is presented in Figure 4, where dashed lines depict connections, which may or may not be present depending on the specific realisation. Connections are not present in Type 1 but are present in Type 2. In both cases the architecture consists of a data input block and $J$ pipeline stages, each stage containing a data routing block and a block of processor elements (PEs) wherein the data input block implements the Step 1 of Algorithm 3, data routing blocks are responsible for Step 2.1, and blocks of PEs are for computations of Step 2.2. The two core architecture types mainly differ by the possibility of data exchange between PEs of the same pipeline stage. In the Type 2 core DWT architecture, PEs of a single stage may exchange intermediate data via interconnections, while in the Type 1 core DWT architecture there are no interconnections between the PEs within a pipeline stage and thus the PEs of a single stage do not exchange data during their operation.
In general, many different realisations of data routing blocks and blocks of PEs are possible. Therefore, in one aspect, the invention can be considered to reside in the architectures as they are depicted at the block level (Figures 4, 11, and 12) and as they are described below at the functional level, independent of the precise implementation chosen for the PEs and data routing blocks. However, some practical realisations of the proposed core DWT architectures at the register level are presented by way of example with reference to Figures 5 to 10. These exemplary implementations demonstrate the validity of the invention.

Figure 4 presents the general structure of the Type 1 and Type 2 core DWT architecture. As explained in the foregoing, Type 1 and Type 2 differ only in the lack or presence of interconnection between the PEs within a stage. The data input block of both core DWT architectures may be realized as either word-serial or word-parallel. In the former case the data input block consists of a single (word-serial) input port which is connected to a shift register of length $2^J$ (dashed lined box in Figure 4) having a word-parallel output from each of its cells. In the latter case the data input block comprises $2^J$ parallel input ports. In both cases the data input block has $2^J$ parallel outputs which are connected to the $2^J$ inputs of the data routing block of the first pipeline stage. In Figure 6 an example of a word-parallel data input block is presented, while Figure 7 and 10 present an example of a word-serial data input block.

Type 1 core DWT architecture

The basic algorithm implemented within the Type 1 core DWT architecture is Algorithm 3 with a specific order of implementing Step 2.2. The structure of the $2^J \times 2^J$ matrix $\hat{D}$ is such that the matrix-vector multiplication of Step 2.2 can be decomposed into $2^J$ pairs of vector-vector inner product computations:

$$\mathbf{x}^{d,s,2^i(i)}(i) = L_P \cdot \hat{\mathbf{x}}^{d,2^i,2^{i+1}(i)}(2i : 2i + L - 1),$$

$$\mathbf{x}^{d,s,2^i(i)}(i + 2^J - i) = H_P \cdot \hat{\mathbf{x}}^{d,2^i,2^{i+1}(i)}(2i : 2i + L - 1),$$

which can be implemented in parallel. On the other hand, every vector-vector inner product of length $L$ can be decomposed into a sequence of \( L_p = \lceil L/p \rceil \) inner products of length $p$ with accumulation of the results (assuming that the coefficient vectors and input vectors are appended with an appropriate number of zeros and are divided into sub-vectors of consecutive $p$ components). As a result, Algorithm 3 can be presented with the following modification to the previous pseudocode.
Algorithm 3.1.

1. For $s = 0, ..., 2^{m-j} - 1$ set $x^{(0,0)}_{LP} = x(s, 2^j : (s + 1) - 2^j - 1)$;

2. For $s = s*(l), ..., 2^{m-j} + s*(J) - 1$

For $j = J_1, ..., J_2$ do in parallel

Begin

2.1. Set $\hat{x}^{(j-s)*n^0}$ according to (4)

2.2. For $i = 0, ..., 2^{J-j} - 1$ do in parallel

Begin

Set $S_{LP}(i) = 0$, $S_{HP}(i) = 0$;

For $n = 0, ..., L_p - 1$ do in sequential

Begin

$$S_{LP}(i) = S_{LP}(i) + \sum_{k=0}^{2^i-1} l_{np+k} \hat{x}^{(6-L-j-s^0 n)}(2i + np + k)$$ (6)

$$S_{HP}(i) = S_{HP}(i) + \sum_{k=0}^{2^i-1} h_{np+k} \hat{x}^{(6-L-j-s^0 n)}(2i + np + k)$$ (7)

End

Set $x^{(6-L-j-s^0 n)}_{LP}(i) = S_{LP}(i)$; $x^{(6-L-j-s^0 n)}_{HP}(i) = S_{HP}(i)$

End

End

3. Form the output vector

$$y = \left[ \left(x^{(6-L-j-s^0 n)}_{LP} \right)^T \ldots \left(x^{(6-L-j-s^0 n)}_{LP} \right)^T \left(x^{(6-L-j-s^0 n)}_{HP} \right)^T \ldots \left(x^{(6-L-j-s^0 n)}_{HP} \right)^T \left(x^{(5-L-j-s^0 n)}_{HP} \right)^T \ldots \left(x^{(5-L-j-s^0 n)}_{HP} \right)^T \right]^T.$$
architecture consists of a data input block (already described above) and J pipeline stages. In general, the jth pipeline stage, \( j = 1, \ldots, J \), of the Type 1 core DWT architecture comprises a data routing block having \( 2^{2j-1} \) inputs \( l_{PSY}(0), \ldots, l_{PSY}(2^{2j-1} - 1) \) forming the input to the stage, and \( 2^{2j-1} + p - 2 \) Outputs \( O_{DRB}(0), \ldots, O_{DRB}(2^{2j-1} + p - 3) \) connected to the inputs of \( 2^{2j} \) PEs. Every PE has \( p \) inputs and two outputs where \( p \leq L_{\text{max}} \) is a parameter of the realisation describing the level of parallelism of every PE. Consecutive \( p \) outputs \( O_{DRB}(2i), O_{DRB}(2i + 1), \ldots, O_{DRB}(2i + p - 1) \) of the data routing block of the \( j \)th, \( j = 1, \ldots, J \), stage are connected to the \( p \) inputs of the \( j \)th PE (PE\(_{j,i}\)) of the same stage. The first outputs of each of \( 2^{2j} \) PEs of the \( j \)th pipeline stage, \( j = 1, \ldots, J - 1 \), form the outputs \( O_{PSY}(0), \ldots, O_{PSY}(2^{2j} - 1) \) of that stage and are connected to the \( 2^{2j} \) inputs \( l_{PSY}(1), \ldots, l_{PSY}(2^{2j} - 1) \) of the data routing block of the next, \( (j+1) \)st, stage. The first output of the (one) PE of the last, \( J \)th, stage is the 0th output \( out(0) \) of the architecture. The second outputs of the \( 2^{2j} \) PEs of the \( j \)th pipeline stage, \( j = 1, \ldots, J \), form the \( (2^{2j}) \) th to \( (2^{2j+1} - 1) \) st outputs \( out(2^{2j}), \ldots, out(2^{2j+1} - 1) \) of the architecture.

[0044] The blocks of the Type 1 core DWT architecture are now described at the functional level. For convenience, a time unit is defined as the period for the PEs to complete one operation (which is equal to the period between successive groups of \( p \) data entering the PE) and an operation step of the architecture is defined as comprising \( L_p \) time units.

[0045] The data input block serially or parallelly accepts and parallelly outputs a group of components of the input vector at the rate of \( 2^m \) components per operation step. Thus, the vector \( X^{(0)}_{LP}(\cdot) \) is formed at the outputs of the data input block at step \( s = 0, \ldots, 2^{m-1} - 1 \).

[0046] The data routing block (of stage \( j = 1, \ldots, J \)) can, in general, be realized as an arbitrary circuitry which at the first time unit \( n = 0 \) of every operation step serially accepts a vector of \( 2^{j+1} \) components, and then at each time unit \( n = 0, \ldots, 2^{m-1} - 1 \) of that operation step it serially outputs a vector of \( 2^{j+1} \) components \( n \cdot p \), where \( p \) is a parameter of the realisation describing the level of parallelism of each PE. The extra delay is a consequence (cost) of the flexibility of the architecture that enables it to implement DWTs with arbitrary filter length \( L \leq L_{\text{max}} \). This should be compared with Algorithm 3.1, which presents computation of a DWT with a fixed filter length \( L_{\text{max}} \). In fact, the architecture is designed for the filter length \( L_{\text{max}} \), but also implements DWTs with shorter filters with a slightly increased time delay but without an increase in time period.

Denote

\[
\hat{Q}_j = \left[ \frac{L_{\text{max}} - 2}{2^{j-1}} \right] \quad j = 1, \ldots, J \quad (8)
\]

[0047] The functionality of the PEs used in the Type 1 core DWT architecture is to compute two inner products of the vector on its \( p \) inputs with two vectors of predetermined coefficients during every time unit and to accumulate (separately) the results of both inner products computed during one operation step. At the end of every operation step, the two accumulated results pass to the two outputs of the PE and a new accumulation starts. Clearly, every PE implements computations according to Algorithm 3.1. In the case where \( L < L_{\text{max}} \) an extra delay is introduced. The extra delay is a consequence (cost) of the flexibility of the architecture that enables it to implement DWTs with arbitrary filter length \( L \leq L_{\text{max}} \). This should be compared with Algorithm 3.1, which presents computation of a DWT with a fixed filter length \( L_{\text{max}} \). In fact, the architecture is designed for the filter length \( L_{\text{max}} \), but also implements DWTs with shorter filters with a slightly increased time delay but without an increase in time period.

Denote

\[
\hat{s}(0) = 0, \quad \hat{s}(j) = \sum_{n=1}^{j} \hat{Q}_{n,j-1}, \quad j = 1, \ldots, J \quad (9)
\]

[0048] During operation of the architecture, the vector \( X^{(0)}_{LP}(\cdot) \) is formed at the outputs of the data input block at step \( s = 0, \ldots, 2^{m-1} - 1 \) and this enters to the inputs of the data routing block of the first pipeline stage. To show that the architecture implements computations according to Algorithm 3.1 it is sufficient to show that the vectors \( X^{(1,2)}_{LP}(\cdot) \) are formed at the first outputs of PEs of the \( j \)th stage (which are connected to the inputs of the \( (j+1) \)st stage) and the vectors \( X^{(1,2)}_{HP}(\cdot) \) are formed at their second outputs at steps \( s = \hat{s}(j), \ldots, \hat{s}(j) + 2^{m-1} - 1 \) provided that the vectors \( X^{(1,2)}_{LP}(\cdot) \) enter to the \( j \)th stage at steps \( s = \hat{s}(j-1), \ldots, \hat{s}(j) + 2^{m-1} - 1 \) (proof by mathematical induction). Thus, it is assumed that the data routing
block of stage $j = 1, \ldots, J$, accepts vectors $\mathbf{x}_{LP}^{\{j-1, s-2^j-1\}}$ at steps $s = \hat{s}(j-1), \ldots, \hat{s}(j-1)+2^{m-J}$. Then, according to the functional description of the data routing blocks, the components $np, np+1, \ldots, (n+1)p+2^{j+1}-3$ of the vector 

\[
\mathbf{x}_{LP}^{\{j-1, s\}} = \begin{bmatrix}
\mathbf{x}_{LP}^{\{j-1, s-2^j-1\}} \\
\mathbf{x}_{LP}^{\{j-1, (s-2^j-1)+2^m-1\}} \\
\mathbf{x}_{LP}^{\{j-1, (s-2^j-1)+2^m-1\}} \\
\mathbf{x}_{LP}^{\{j-1, (s-2^j-1)+2^m-1\}} \\
\end{bmatrix}
\]  

(10)

which is the concatenation of the vectors accepted at steps $s, s+2, \ldots, s$, respectively, will be formed on the outputs of the data routing block at the time unit $n = 0, \ldots, L_p-1$ of every step $s = \hat{s}(j), \ldots, \hat{s}(j)+2^{m-J}$. Since $\hat{s}(j) \geq s^*(j)$ (compare (3) and (9)), the vector $\mathbf{x}_{LP}^{\{j-1, s\}}$ (defined according to equation (4)) is the sub-vector of $\mathbf{x}_{LP}^{\{j-1, s\}}$ so that their first $2^{j+1} + L-3$ components are exactly the same. Thus the vector $\mathbf{x}_{LP}^{\{j-1, s\}} = \mathbf{x}_{LP}^{\{j-1, s\}}(np, 2^{j+1} + (n+1)p-3)$ is formed at the time unit $n = 0, \ldots, L_p-1$ of step $s = \hat{s}(j), \ldots, \hat{s}(j)+2^{m-J}$ at the outputs of the data routing block of stage $j = 1, \ldots, J$. Due to the connections between the data routing block and PEs, the components $2i, 2i+1, \ldots, 2i+p-1$ of the vector $\mathbf{x}_{LP}^{\{j-1, s\}}$ which are, in fact, arguments of the operations (6) and (7), will be formed on the inputs of the PE$_{j,i}$, $i = 0, \ldots, 2^j-1$ at time unit $n = 0, \ldots, L_p-1$ of step $s = \hat{s}(j), \ldots, \hat{s}(j)+2^{m-J}$. Thus, if the PEs implement their operations with corresponding coefficients, the vector $\mathbf{x}_{LP}^{\{j, s\}}$ will be formed at the first outputs of the PEs and the vector $\mathbf{x}_{HP}^{\{j, s\}}$ will be formed at their second outputs after step $s = \hat{s}(j), \ldots, \hat{s}(j)+2^{m-J}$. Since the first outputs of PEs are connected to the inputs of the next pipeline stage this proves that the architecture implements computations according to the Algorithm 3.1, albeit with different timing (replace $s^*(j)$ with $\hat{s}(j)$ everywhere in Algorithm 3.1).

From the above considerations it is clear that a $2^m$-point DWT is implemented with the Type 1 core DWT architecture in $2^{m-J} \hat{s}(J)$ steps each consisting of $L_p$ time units. Thus, the delay between input and corresponding output vectors is equal to

\[
T_d(C1) = [2^m - j + \hat{s}(J)][L / p]
\]

(11)
time units. Clearly the architecture can implement DWTs of a stream of input vectors. It is therefore apparent that the throughput or the time period (measured as the interval between time units when successive input vectors enter the architecture) is equal to

\[
T_p(C1) = 2^{m-J} [L / p]
\]

(12)
time units.

Performance of parallel/pipelined architectures is often evaluated with respect to hardware utilization or efficiency, defined as

\[
E = \frac{T(1)}{K \cdot T(K)} \cdot 100 \%
\]

(13)

where $T(1)$ is the time of implementation of an algorithm with one PE and $T(K)$ is the time of implementation of the same algorithm with an architecture comprising $K$ PEs. It can be seen that $T(1) = (2^m - 1)L/p$ time units are required to implement a $2^m$-point DWT using one PE similar to the PEs used in the Type 1 core DWT architecture. Together with
(11) and (12), and taking into account the fact that there are in total $K = 2^J - 1$ PEs within the Type 1 core DWT architecture, it can be shown that approximately 100% efficiency (hardware utilisation) is achieved for the architecture both with respect to time delay or, moreover, time period complexities. It should be noted that an efficiency close to the efficiency of the FPP architecture is reached only in a few pipelined DWT designs known from prior art (see [17]), whereas most of the known pipelined DWT architectures reach much less than 100% average efficiency. It should also be noted that a time period of at least $O(N)$ time units is required by known DWT architectures. The proposed architecture may achieve a desired time period as it may be realized with a varying level of parallelism depending on the parameter $p$. As follows from (12), the time period complexity of the implementation varies between $T_1(C_1) = 2^m - J$ and $T_1(C_1) = 2^m - J$.

Thus, the throughput of the architecture is $2^{j} / L$ to $2^{j}$ times faster than that of the fastest known architectures.

The possibility of realising the architecture with a varying level of parallelism also gives an opportunity to trade-off time and hardware complexities. It should also be noted that the architecture is very regular and only requires simple control structures (essentially, only a clock) unlike, e.g. the architecture of [17]. It does not contain a feedback, switches, or long connections that depend on the size of the input, but only has connections which are at maximum only $O(L)$ in length. Thus, it can be implemented as a semisystolic array.

A possible realisation of the Type 1 core DWT architecture

A possible structure of the $j$th pipeline stage, $j = 1, \ldots, J$, for the Type 1 core DWT architecture is depicted in Figure 5. Two examples of such realisation for the case $L_{\text{max}} = 6$, $J = 3$ are shown in Figures 6 and 7, where $p = L_{\text{max}} = 6$ and $p = 2$, respectively. It should be noted that a particular case of this realisation corresponding to the case $p = L_{\text{max}}$ and, in particular, a specific version of the example in Figure 6, was presented in publications [34] and [35], where it was referred to as a limited parallel-pipelined (LPP) architecture. In contrast, the Type 1 core DWT architecture and its realisation in Figure 5 are for the case of arbitrary $p$. It should further be noted that the LPP architecture described in documents [34] and [35] does not support efficient computation of DWTs with filter lengths less than $L_{\text{max}}$, while the realisation of the Type 1 core DWT architecture presented in Figure 5 does.

A possible realisation of the Type 1 core DWT architecture

A possible realisation of the Type 1 core DWT architecture presented in Figure 5 does.

A possible structure of the Type 2 core DWT architecture

The Type 2 core DWT architecture implements a slightly modified version of Algorithm 3.1. The modification is based on the observation that operands of operations (6) and (7) are the same for pairs $(i_1, n_1)$ and $(i_2, n_2)$ of indices $i$ and $n$ such that $2i_1 + n_1 p = 2i_2 + n_2 p$. Assuming an even $p$ (the odd case is treated similarly but requires more notation for its representation), this means that, when implementing the operations of (6) and (7), the multiplicands required for use in time unit $n = 1, \ldots, L_p - 1$ within branch $i = 0, \ldots, 2^{j-i} - 1$ can obtained from the multiplicands obtained at step $n - 1$ within branch $i + p / 2$. The corresponding computational process is described with the following pseudocode where we denote:

$$\text{EP 1 412 911 B1}$$
Algorithm 3.2.

1. For \( s = 0, \ldots, 2^m - 1 \) set \( x_L^{(0,s)} = x(s \cdot 2^j : (s+1) \cdot 2^j - 1) \);

2. For \( s = s^*(0), \ldots, 2^m - s^*(J) - 1 \)

For \( j = J_1, \ldots, J_2 \) do in parallel

Begin

2.1. Set \( \hat{x}_{L,J+1}^{(j,1), s^*(j)} \) according to (4)

2.2. For \( i = 0, \ldots, 2^{j-1} - 1 \) do in parallel

Begin

For \( k = 0, \ldots, p - 1 \)

Begin

set \( z_{L_P}(i, 0, k) = l_k \hat{x}_{L,J+1}^{(j,1), s^*(j)}(2i + k) \);

set \( z_{H_P}(i, 0, k) = h_k \hat{x}_{L,J+1}^{(j,1), s^*(j)}(2i + k) \)

Compute \( S_{L_P}(i) = \sum_{k=0}^{p-1} z_{L_P}(i, 0, k) ; \quad S_{H_P}(i) = \sum_{k=0}^{p-1} z_{H_P}(i, 0, k) ; \)

End

For \( n = 1, \ldots, L_p - 1 \) do in sequential

Begin

For \( k = 0, \ldots, p - 1 \)

Begin

set \( z_{L_P}(i, n, k) = \begin{cases} 
   l'_k + \frac{n}{2} x_{L,J+1}^{(j,1), s^*(j)}(2i + k) & \text{if } i < 2^{j-1} - p/2 \\
   l'n + k & \text{if } i \geq 2^{j-1} - p/2 
   \end{cases} \)

set \( z_{H_P}(i, n, k) = \begin{cases} 
   h'_k + \frac{n}{2} x_{L,J+1}^{(j,1), s^*(j)}(2i + k) & \text{if } i < 2^{j-1} - p/2 \\
   h'n + k & \text{if } i \geq 2^{j-1} - p/2 
   \end{cases} \)

End

Compute \( S_{L_P}(i) = S_{L_P}(i) + \sum_{i=0}^{p-1} z_{L_P}(i, n, k) ; \quad S_{H_P}(i) = S_{H_P}(i) + \sum_{k=0}^{p-1} z_{H_P}(i, n, k) ; \)
The general structure of the Type 2 core DWT architecture is presented in Figure 4. In this case, connections between PEs (the dashed lines) belonging to the same pipeline stage are valid. As follows from Figure 4 the Type 2 core DWT architecture is similar to the Type 1 core DWT architecture but now except for \( p \) inputs and two outputs (later on called main inputs and main outputs) every PE has an additional \( p \) inputs and \( p \) outputs (later on called intermediate inputs and outputs). The \( p \) intermediate outputs of \( PE_{j,p} \) are connected to the \( p \) intermediate inputs of \( PE_{j,p} \). The other connections within the Type 2 core DWT architecture are similar to those within the Type 1 core DWT architecture.

The functionality of the blocks of the Type 2 core DWT architecture are substantially similar to those of the Type 1 core DWT architecture. The functionality of the data input block is exactly the same as for the case of the Type 1 core DWT architecture.

The data routing block (of the stage \( j = 1, \ldots, J \)) can, in general, be realized as an arbitrary circuitry which at the first time unit \( n = 0 \) of every operation step accepts a vector of \( 2^j \times 1 \) components in parallel, and parallelly outputs a vector of the first \( 2^j+1 \) components of a vector which is the concatenation (in chronological order) of the vectors accepted at the previous \( Q \) steps, where \( Q \) is defined in (8). Then at every time unit \( n = 0, \ldots, L_{\max} \) of that operation step, the data routing block parallelly outputs the next sub-vector of \( p \) components \( 2^j+1 \times p \) of the same vector on its last \( p \) outputs.

The functionality of the PEs used in the Type 2 core DWT architecture at every time unit \( n = 0, \ldots, L_{\max} \) of every operation step is to compute two inner products of a vector \( x \), present on either of its \( p \) main or \( p \) intermediate inputs with two vectors of predetermined coefficients, \( LP' \) and \( HP' \) of length \( p \), as well as to compute a point-by-point product of \( x \) with \( LP' \). At time unit \( n = 0 \) the vector \( x \) is formed using the main \( p \) inputs of the PE and at time units \( n = 1, \ldots, L_{\max} \) vector \( x \) is formed using the intermediate inputs of the PE. Results of both inner products computed during one operation step are accumulated and are passed to the main outputs of the PE, while the results of the point-by-point products are passed to the intermediate outputs of the PE.

Similar to the case of the Type 1 core DWT architecture, it can be seen that the Type 2 core DWT architecture implements Algorithm 3.2 with time delay and time period characteristics given by (11) and (12). The other characteristics of the Type 1 and Type 2 architectures are also similar. In particular, the Type 2 architecture is very fast, may be implemented as a semisystolic architecture and with a varying level of parallelism, providing an opportunity for creating a trade-off between time and hardware complexities. A difference between these two architectures is that the shift registers of the data routing blocks in the Type 1 core DWT architecture are replaced with additional connections between PEs within the Type 2 core DWT architecture.

A possible realisation of the Type 2 core DWT architecture

A possible structure of the \( j \)th pipeline stage, \( j = 1, \ldots, J \), for the Type 2 core DWT architecture is depicted in Figure 9. An example of such a realisation for the case \( L_{\max} = 6 \), \( J = 3 \) and \( p = 2 \) is shown in Figure 10(a). In this realisation the data routing block consists of \( Q \) chain connected groups of \( 2^j+1 \) delays each, and a shift register of length \( L_{\max}-2 \) which shifts the values upwards by \( p \) positions every time unit. The \( 2^j+1 \) inputs to the stage are connected in parallel to the first group of delays, the outputs of which are connected to the inputs of the next group of delays etc. The outputs of the last \( Q \)th group of delays form the first \( 2^j+1 \) outputs of the data routing block and are connected to the main inputs of the PEs. The outputs of the \( (Q_j-1) \)th group of delays, \( t = 1, \ldots, Q_j \), are connected in parallel to the \( 2^j+1 \) consecutive cells of the shift register. The outputs of the \( (Q_j-1) \)st group of delays are connected to the first \( 2^j+1 \) cells, the outputs of the \( (Q_j-2) \)nd group of delays are connected to the next \( 2^j+1 \) cells etc. However, the first \( q_j = (L_{\max}-\)
2) \((Q_j-1)2^{2j+1}\) inputs of the stage are directly connected to the last \(Q_j\) cells of the shift register. The outputs from the first \(p-2\) cells of the shift register form the last \(p-2\) outputs of the data routing block and are connected to the main inputs of the PEs according to the connections within the general structure. It can be shown that the presented realization satisfies the functionality constraint for the data routing block of the Type 2 core DWT architecture. Indeed, at the beginning of every step, the first \(2^{2j+1}+p-2\) components of a vector which is the concatenation (in chronological order) of the vectors accepted at previous \(Q_j\) steps are formed at the outputs of the data routing block and then during every following time unit the next \(p\) components of that vector are formed at its last \(p\) outputs.

[0062] A possible PE structure for the Type 2 core DWT architecture for the case of \(p = 2\) is presented in Figure 10 (b). It will be apparent to one of ordinary skill in the art that structures for arbitrary \(p\) and for \(p = 1, p = 2,\) and \(p = L_{\text{max}}\) can be designed similar to those illustrated in Figures 8(a), (b), (c), and (d).

[0063] It should be noted that in the case \(p = L_{\text{max}}\) this realisation of the Type 2 core DWT architecture is the same as the realisation of the Type 1 core DWT architecture depicted in Figure 6.

Multi-core DWT architectures

[0064] The two types of core DWT architectures described above may be implemented with varying levels of parallelism depending on the parameter \(p\). Further flexibility in the level of parallelism is achieved within multi-core DWT architectures by introducing a new parameter \(r = 1, ..., 2^{m-2}\). The multi-core DWT architecture is, in fact obtained from a corresponding single-core DWT architecture by expanding it \(r\) times. Its general structure is presented in Figure 11. The architecture consists of a data input block and \(J\) pipeline stages each stage containing a data routing block and a block of PEs.

[0065] The data input block may be realized as word-serial or as word-parallel in a way similar to the case of core DWT architectures, but in this case it now has \(r^2\) parallel outputs which are connected to the \(r^2\) inputs of the data routing block of the first pipeline stage. The functionality of the data input block is to serially or parallelly accept and parallelly output a group of components of the input vector at the rate of \(r^2\) components per operation step.

[0066] Consider firstly the Type 1 multi-core DWT architecture. In this case, the \(j\)th pipeline stage, \(j = 1, ..., J\), consists of a data routing block having \(2^{2j+1}\) inputs \(I_{PS(j)}(0), ..., I_{PS(j)}(2^{2j+1}-1)\) forming the input to the stage, and \(2^{2j+1}+p-2\) outputs \(O_{DRB(j)}(0), ..., O_{DRB(j)}(2^{2j+1}+p-3)\) connected to the inputs of \(2^{2j}\) PEs. Every PE has \(p\) inputs and two outputs where \(p \leq L_{\text{max}}\) is a parameter of the realisation describing the level of parallelism of every PE. Consecutive \(p\) outputs \(O_{DRB(j)}(2i), O_{DRB(j)}(2i+1), ..., O_{DRB(j)}(2i+p-1)\) of the data routing block of the \(2j\)th, \(j = 1, ..., J\), stage are connected to the \(p\) inputs of the \(i\)th \(PE\) of the same stage. The first outputs of \(2^{2j}\) PEs of the \(j\)th pipeline stage, \(j = 1, ..., J\), form the outputs \(O_{PS(j)}(0), ..., O_{PS(j)}(2^{2j}-1)\) of that stage and are connected to the \(2^{2j}\) inputs \(I_{PS(j+1)}(0), ..., I_{PS(j+1)}(2^{2j}-1)\) of the data routing block of the next, \((j+1)\)st, stage. The first outputs of the \(r\) PEs of the last, \(J\)th, stage form the first \(r\) outputs \(out(0), ..., out(r-1)\) of the architecture. The second outputs of the \(2^{2j}\) PEs of the \(j\)th pipeline stage, \(j = 1, ..., J\), form the \((2^{2j})th\) to \((2^{2j+1}-1)th\) st outputs \(out(2^{2j}), ..., out(2^{2j+1}-1)\) of the architecture.

[0067] The data routing block of stage (of stage \(j = 1, ..., J\)) can, in general, be realized as an arbitrary circuitry which at the first time unit \(n = 0\) of every operation step parallelly accepts a vector of \(2^{2j+1}\) components, and then at every time unit \(n = 0, ..., L_r - 1\) of that operation step it parallelly outputs a vector of \(2^{2j+1}\) components \(p = 2\) components \(np, np+1, ..., (n+1)p+2^{2j+1}-1\) of a vector which is the concatenation (in chronological order) of the vectors accepted at the previous \(Q_j\) steps (see (8)). The functionality of the PEs is exactly the same as in the case of the Type 1 core DWT architecture.

[0068] Consider now the Type 2 multi-core DWT architecture. The data input block is exactly the same as in the case of the Type 1 multi-core DWT architecture. The PEs used in the Type 2 multi-core DWT architecture and the interconnections between them are similar to the case of the Type 2 single-core DWT architecture. The difference is that now there are \(2^{2j}\) (instead of \(2^{2j}\)) PEs within the \(j\)th pipeline stage, \(j = 1, ..., J\), of the architecture. The data routing block now has \(2^{2j+1}\) inputs and \(2^{2j+1}+p-2\) outputs with similar connections to PEs as in the case of the Type 1 multi-core DWT architecture. The data routing block of stage \(j = 1, ..., J\) can, in general, be realized as an arbitrary circuitry which at the first time unit \(n = 0\) of every operation step parallelly accepts a vector of \(2^{2j+1}\) components, and parallelly outputs a vector of the first \(2^{2j+1}+p-2\) components \(0, ..., 2^{2j+1}+p-3\) of a vector which is the concatenation (in chronological order) of the vectors accepted at the previous \(Q_j\) steps. Then at every time unit \(n = 0, ..., L_r - 1\) of that operation step it parallelly outputs the next sub-vector of \(p\) components \(2^{2j+1} + np - 2, ..., 2^{2j+1} + (n+1)p - 3\) of the same vector on its last \(p\) outputs.

[0069] Both types of multi-core DWT architectures are \(r\) times faster than the single-core DWT architectures, that is a linear speed-up with respect to the parameter \(r\) is achieved. The delay between input and corresponding output vectors is equal to

\[
T_d(Ci) = \left\lfloor \frac{x_{2j} + i(j)\times L!}{p} \right\rfloor r
\]
time units and the throughput or the time period is equal to

\[ T_p(C1) = 2^{m-J} [L/p] r \quad \text{.. (15)} \]

time units. Thus, further speed-up and flexibility for trade-off between time and hardware complexities is achieved within multi-core DWT architectures. In addition, the architectures are modular and regular and may be implemented as semi-systolic arrays.

As a possible realisation of the multi-core DWT architecture for the case of \( p = L = L_{\text{max}} \) and \( r = 2^{m-J} \) the DWT flowgraph itself (see Figure 2) may be considered where nodes (rectangles) represent PEs and small circles represent latches. This realization was presented in [34], where it was referred to as fully-parallel pipelined (FPP) architecture. However, it is only a specific realisation of the multi-core DWT architecture proposed according to the present invention.

The variable resolution DWT architecture

The architectures described above implement DWTs with a number of octaves not exceeding a given number \( J \). They may implement DWTs with a number of octaves smaller than \( J \), albeit with some loss in hardware utilisation. The variable resolution DWT architecture implements DWTs with an arbitrary number of octaves \( J' \) while the efficiency of the architecture remains approximately 100% whenever \( J' \) is larger than or equal to a given number \( J_{\text{min}} \).

The general structure of the variable resolution DWT architecture is shown in Figure 12(a). It consists of a core DWT architecture corresponding to \( J_{\text{min}} \) decomposition levels and an arbitrary serial DWT architecture, for instance, one based on an RPA ([14]-[17], [19]-[20], [22]). The core DWT architecture implements the first \( J_{\text{min}} \) octaves of the \( J' \)-octave DWT. The low-pass results from the output (out\( (0) \)) of the core DWT architecture are passed to the serial DWT architecture. The serial DWT architecture implements the last \( J' - J_{\text{min}} \) octaves of the \( J' \)-octave DWT. Since the core DWT architecture may be implemented with a varying level of parallelism, it can be balanced with the serial DWT architecture in such a way that approximately 100% hardware utilisation is achieved whenever \( J' \geq J_{\text{min}} \).

To achieve a balance between the two parts, the core DWT architecture is arranged to implement a \( J_{\text{min}} \)-octave \( N \)-point DWT with a throughput which is the same or faster than the serial architecture implements a \( (J' - J_{\text{min}}) \)-octave \( M \)-point DWT (\( M = (N/2^{J_{\text{min}}}) \)). Serial architectures found in the literature implement an \( M \)-Point DWT either in \( 2M \) time units ([14], [15]) or in \( M \) time units ([14]-[19]) correspondingly employing either \( L \) or \( 2L \) basic units (BUs, multiplier-adder pairs). They can be scaled down to contain an arbitrary number \( K \leq 2L \) of BUs so that an \( M \)-point DWT is implemented in \( M \cdot 2L/K \) time units. Since the Type 1 or Type 2 core DWT architectures implement a \( J_{\text{min}} \)-octave \( N \)-point DWT in \( N[L/p]/2^{J_{\text{min}}} \) time units, the balancing condition becomes \( \Gamma L/p \leq 2L/K \) which will be satisfied if \( p = \Gamma K/2L \). With this condition the variable resolution DWT architecture consists of a total number

\[ A = 2p(2^{J_{\text{min}}} - 1) + K = \begin{cases} K2^{J_{\text{min}}} , & \text{if } K \text{ is even} \\ (K+1)2^{J_{\text{min}}} - 1 , & \text{if } K \text{ is odd} \end{cases} \]

of BUs and implements a \( J' \)-octave \( N \)-point DWT in

\[ T_d = N[2L/K]/2^{J_{\text{min}}} \]

time units.

A variable resolution DWT architecture based on a multi-core DWT architecture may also be constructed (see Figure 12(b)) with a data routing block inserted between the multi-core and serial DWT architectures. The functionality of the data routing block is to parallelly accept and serially output digits at the rate of \( r \) samples per operation step. The balancing condition in this case is \( rp = \lceil K/2 \rceil \), and the area time characteristics are
Table 1, presented in Figure 14, compares the performance of the proposed architectures with some conventional architectures. In this table, as commonly accepted in the literature, the number of multiplier-adder pairs (which are the basic units (BUs) in DWT architectures) is considered to be representative of the area of a given architecture. The time unit is taken to be the time period required by one multiplication since this is the critical pipeline stage. Characteristics of the DWT architectures proposed according to the invention, shown in the last seven rows of Table 1, are given for arbitrary realisation parameters $L_{\text{max}}$, $p$, and $r$ as well as for some examples of parameter choices. It should be mentioned that the numbers of BUs used in the proposed architectures are given assuming the PE examples of Figure 8 (where a PE with $p$ inputs contains $2^p$ BUs). However, the PEs could be further optimized to involve a smaller number of BUs.

For convenience, Table 2, shown in Figure 15, presents numerical examples of area-time characteristics for the choice of the DWT parameters $J = 3$ or $J = 4$, $N = 1024$, and $L = 9$ (This corresponds to the most popular DWT, the Daubechies 9/7 wavelet). Table 3, shown in Figure 16, presents numerical examples for the case $J = 3$ or $J = 4$, $N = 1024$, and $L = 5$ the Daubechies 5/3 wavelet). The gate counts presented in the tables were calculated assuming that a BU consists of a 16-bit Booth multiplier followed by a hierarchical 32-bit adder and thus involves a total of 1914 gates (see [37]). Figure 13 represents some of the rows from Table 2 in graphical form. It should be noted that the line corresponding to the proposed architectures may be extended to much larger numbers of basic units, although these non-present cases require rather large silicon areas, which might be impractical at the current state of the technology.

As follows from these illustrations, the proposed architectures, compared to the conventional ones, demonstrate excellent time characteristics at moderate area requirements. Advantages of the proposed architectures can be best appreciated when considering their performance with respect to the $AT_p^2$ criterion, which is commonly used to estimate the performance of high-speed orientated architectures. Note that the first row of the Tables represent a general purpose DSP architecture. Architectures presented in the next two rows are either non-pipelined or restricted (only two stage) pipelined architectures which operate at approximately 100% hardware utilisation as do the architectures proposed according to the invention. Thus, their performance is "proportional" to the performance of the proposed architectures which, however, are much more flexible in the level of parallelism that can be achieved, resulting in a wide range of possible time and area complexities. The fourth row of the tables presents $J$ stage pipelined architectures with poor hardware utilisation and consequently a poor performance. The fifth to seventh rows of the tables present architectures from previous publications which are $J$ stage pipelined and achieve 100% hardware utilisation and good performance but do not allow a flexible range of area and time complexities such as that provided by the architectures proposed according to the invention.

In the foregoing there has been discussion of general structures of "universal" wavelet transformers which are able to implement the wavelet transform with arbitrary parameters such as filter lengths and coefficients, input length, and the number of decomposition levels. Further optimisation of the architectures for a specific discrete wavelet transform (corresponding to a specific set of the above parameters) is possible by optimizing the structure of processing elements (PEs) included in the architecture.

The invention can be implemented as a dedicated semisystolic VLSI circuit using CMOS technology. This can be either a stand-alone device or an embedded accelerator for a general-purpose processor. The proposed architectures can be implemented with varying level of parallelism which leads to varying cost and performance. The choice of the mode of implementation, as well as the desired level of parallelism depends on the application field. In the foregoing, the architectures were described in the context of implementations for performing arbitrary DWT. However, they can be further optimized and implemented in such a way as to be dedicated to a specific type of DWT. This may be desirable in applications relating to JPEG 2000, for example, where Daubechies 5/3 or 9/7 wavelets are planned to be the basic DWTs.

### Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
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<tbody>
<tr>
<td>ASIC</td>
<td>Application Specific Integrated Circuits</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Silicon</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
</tr>
</tbody>
</table>
DWT          Discrete Wavelet Transform  
FPP          Fully Parallel-Pipelined (DWT architecture)  
LPP          Limited Parallel-Pipelined (DWT architecture)  
PE           Processor element  

References  

[0081]  


[36] http://www.ti.com/sc/docs/products/dsp/c6000/benchmarks/64x.htm#fifters

Claims

1. A core processing circuit for a microprocessor, the core processing circuit arranged to perform a discrete wavelet transform operation, using k different filtering operations, over consecutive decomposition levels, the core processing circuit comprising J consecutive processing stages j, each one of said J consecutive processing stages j corresponding to a decomposition level of the discrete wavelet transform and being implemented with at least one basic processing circuit element (PE) at each processing stage, said at least one basic processing circuit element (PE) being arranged to perform respective inner product calculations associated with the k different filtering operations of the discrete wavelet transform operation, the number of basic processing circuit elements (PE) implemented in each of said J consecutive processing stages decreasing by a constant factor with each increasing decomposition level j.

2. A core processing circuit according to claim 1, wherein said constant factor is k.

3. A core processing circuit according to claim 1, wherein one of said k different filtering operations is a low-pass filtering operation.

4. A core processing circuit according to claim 3, wherein each processing stage j = 1 to J comprises a data routing block coupled to the respective basic processing circuit elements (PE) of the processing stage, the data routing block of each respective processing stage j having k^{j-1} \cdot J parallel inputs and being arranged to route sample values received at said parallel inputs to the basic processing circuit elements (PE) of the respective processing stage, the data routing block of the first processing stage, j = 1, being arranged to receive in parallel k^1 input sample values from a data input block and each of the data routing blocks of the subsequent processing stages, j = 2 to J, being arranged to receive in parallel k^{j-1} \cdot J output sample values from the low-pass filtering operation of an immediately preceding processing stage.

5. A core processing circuit according to claim 4, wherein the data input block is arranged to:
   - receive k^1 input samples of an input vector x in parallel; and
   - provide said k^1 input samples in parallel to the k^1 parallel inputs of the data routing block of the first processing stage, j = 1.

6. A core processing circuit according to claim 4, wherein the data input block is arranged to:
   - receive k^1 input samples of an input vector x in series; and
   - provide said k^1 input samples in parallel to the k^1 parallel inputs of the data routing block of the first processing stage, j = 1.

7. A core processing circuit according to claim 4, wherein the data routing block of a particular processing stage is arranged to provide all the basic processing circuit elements (PE) of the particular processing stage with respective groups of p input values in parallel.

8. A core processing circuit according to claim 7, wherein the basic processing circuit elements (PE) of the particular processing stage are arranged to operate in parallel on their respective groups of p input values to perform respective inner product operations of length p.

9. A core processing circuit according to claim 8, wherein the basic processing circuit elements (PE) of the particular processing stage are arranged to operate in parallel on consecutive groups of p input values received in parallel to each produce, in parallel, one respective output value for each of said k different filtering operations.

10. A core processing circuit according to claim 4, wherein the data routing block of processing stage j is arranged to:
    - concatenate consecutive groups of k^{j-1} \cdot J output values received in parallel from the low-pass filtering operation of an immediately preceding processing stage j - 1 in consecutive operating steps to form an input vector comprising input values for the basic processing circuit elements (PE) of processing stage j, the amount of time taken by the basic processing circuit elements (PE) of processing stage j - 1 to produce a group of k^{j-1} \cdot J output values in parallel defining an operating step of the core processing circuit; and
    - output a respective group of p input values from said input vector to each basic processing circuit element.
11. A core processing circuit according to claim 10, wherein the data routing block comprises a shift register arranged to concatenate said consecutive groups of \( k^{j-1} \) output values received in consecutive operating steps from the low pass filtering operation of said immediately preceding processing stage \( j-1 \).

12. A core processing circuit according to claim 10, wherein each basic processing circuit element (PE) of each processing stage \( j \) has \( p \) inputs arranged to receive \( p \) input values in parallel and is arranged to perform \( k \) different filtering operations of length \( L \) by applying \( k \) respective sets of \( L \) coefficient values to a set of \( \lceil L/p \rceil \) consecutive parallel groups at said \( p \) inputs during consecutive time units, where \( \lceil \cdot \rceil \) represents rounding up to the nearest integer, and wherein each basic processing circuit element (PE) comprises \( k \) outputs, each output arranged to output an output value corresponding to a result of one of said \( k \) different filtering operations during each operating step.

13. A core processing circuit according to claim 12, wherein filter length \( L \) is selectable in a range from 1 to \( L_{\text{max}} \).

14. A core processing circuit according to claim 10, wherein each basic processing circuit element (PE) of each processing stage \( j \) has \( p = L_{\text{max}} \) inputs arranged to receive \( L_{\text{max}} \) input values in parallel, \( L_{\text{max}} \) corresponding to a predetermined maximum filter length, and wherein each basic processing circuit element (PE) is arranged to perform \( k \) different filtering operations by applying \( k \) respective sets of \( L_{\text{max}} \) coefficient values to a set of \( L_{\text{max}} \) input values received in parallel at said \( L_{\text{max}} \) inputs in a given time unit, said wherein each basic processing circuit element (PE) comprises \( k \) outputs, each output arranged to output an output value corresponding to a result of one of said \( k \) different filtering operations during each time unit thereby providing an operating step corresponding to a single time unit.

15. A core processing circuit according to claim 14, wherein each basic processing circuit element (PE) is arranged to perform \( k \) different filtering operations of length \( L \) by applying \( k \) respective sets of \( L \) coefficient values, where \( L \leq L_{\text{max}} \), to a set of \( L \) input values received in parallel at said \( L_{\text{max}} \) inputs.

16. A core processing circuit according to any of claims 10 to 15, wherein at least a first basic processing circuit element (PE) in a processing stage \( j \) is arranged to receive a set of \( p \) values representing \( p \) intermediate filtering values obtained in a second basic circuit processing element (PE) of the same processing stage \( j \) in an earlier time unit and said first basic processing circuit element (PE) is arranged to use said set of \( p \) intermediate filtering values in a current time unit.

17. A core processing circuit according to claim 16, wherein said first basic processing circuit element (PE) is arranged to receive said \( p \) intermediate values in parallel.

18. A core processing circuit according to claim 16, wherein said first basic processing circuit element (PE) is arranged to receive a set of \( p \) input values in parallel from said second basic processing circuit element (PE) at each successive time unit, after an initial first time unit.

19. A core processing circuit according to any of claims 1 to 18, wherein each basic processing circuit element (PE) is arranged to perform \( k = 2 \) different filtering operations, wherein one the two different filtering operations is a low-pass filtering operation (LP) and the other of said two different filtering operations is a high-pass filtering operation (HP).

20. A core processing circuit according to claim 10, wherein each basic processing circuit element (PE) of each processing stage \( j \) has \( p \) inputs arranged to receive \( p \) input values in parallel and is arranged to perform two different filtering operations of length \( L \) by applying two respective sets of \( L \) coefficient values to a set of \( \lceil L/p \rceil \) input values received in \( \lceil L/p \rceil \) consecutive parallel groups at said \( p \) inputs during consecutive time units, where \( \lceil \cdot \rceil \) represents rounding up to the nearest integer, a first of said two different filtering operations being a low-pass filtering operation (LP) and a second of said two different filtering operations being a high-pass filtering operation (HP), and wherein each basic processing circuit element (PE) comprises two outputs, a first output arranged to output a low-pass filtered output value corresponding to a result of the low-pass filtering operation (LP) performed on said set of \( L \) input values during each operating step, and a second output being arranged to output a high-pass filtered output value corresponding to a result of the high-pass filtering operation (HP) performed on said set of \( L \) input values during each operating step.
21. A core processing circuit according to claim 20, wherein the core processing circuit is arranged to, at each processing stage $j = 2$ to $J$, receive pairs of output values from respective outputs of the basic processing circuit elements (PE) of an immediately preceding processing stage $j - 1$ and to perform a perfect unshuffle operation on a vector formed from said pairs of output values to group together output values from like filtering operations, said output values from immediately preceding processing stage $j - 1$ to be provided as input values to the data routing block of the current processing stage $j$.

22. A core processing circuit according to claim 10, wherein the core processing circuit is arranged to, at each processing stage $j = 2$ to $J$, receive sets of $k$ output values from the respective outputs of the basic processing circuit elements (PE) of an immediately preceding processing stage $j - 1$, and to perform a stride permutation operation on a vector formed from said sets of $k$ output values to group together output values from like filtering operations, said output values from immediately preceding processing stage $j - 1$ to be provided to the data routing block of the current processing stage $j$.

23. A core processing circuit according to claim 19, wherein the core processing circuit is arranged to produce a discrete wavelet transform result to a decomposition level $j$ by forming a vector from output values of a high-pass filtering operation and a low-pass filtering operation at processing stage $j$ and the output values from high-pass filtering operations of all preceding processing stages.

24. A core processing circuit according to any of claim 1 to 23, integrated in a microprocessor circuit with a second processing circuit for performing a discrete wavelet transform, said second processing circuit being arranged to receive output from the core processing circuit and perform at least one further decomposition level.

25. A core processing circuit according to claim 24, wherein said second processing circuit is arranged to perform a recursive pyramid algorithm.

26. A multi-core processing circuit comprising a plurality $r$ of core processing circuits according to any of claims 1 to 25, the core processing circuits of said plurality $r$ being arranged to operate in parallel on respective sets of $k^j$ input samples.

27. A microprocessor comprising a core processing circuit according to any of claims 1 to 25, or a plurality $r$ of core processing circuits according to claim 26.

28. A method of performing a discrete wavelet transform operation with a core processing circuit using $k$ different filtering operations, over consecutive decomposition levels, the core processing circuit comprising $J$ consecutive processing stages, each one of said $J$ consecutive processing stages corresponding to a decomposition level of the discrete wavelet transform and being implemented with at least one basic processing circuit element (PE) at each processing stage, said at least one basic processing circuit element (PE) being arranged to perform respective inner product calculations associated with the $k$ different filtering operations of the discrete wavelet transform operation, the method comprising portioning the input signal vector into consecutive portions of $k^j$ values and using the core processing circuit for each consecutive portion.

29. A method of performing a discrete wavelet transform operation with a multi-core processing circuit, using $k$ different filtering operations, over consecutive decomposition levels, the multi-core processing circuit comprising a plurality $r$ of core processing circuits, each core processing circuit comprising $J$ consecutive processing stages, each one of said $J$ consecutive processing stages corresponding to a decomposition level of the discrete wavelet transform and being implemented with at least one basic processing circuit element (PE) in each core processing circuit at each processing stage, each of said basic processing circuit elements (PE) being arranged to perform respective inner product calculations associated with the $k$ different filtering operations of the discrete wavelet transform operation, the method comprising portioning the input signal vector into consecutive portions of $r \cdot k^j$ values and using said plurality $r$ of core processing circuits in parallel on consecutive portions of $r \cdot k^j$ values.

Patentansprüche

1. Kern-Verarbeitungsschaltung für einen Mikroprozessor, wobei die Kern-Verarbeitungsschaltung eingerichtet ist, einen diskreten Wavelet-Transformations-Vorgang unter Verwendung von $k$ verschiedenen Filtervorgängen über aufeinander folgende Zerlegungsebenen auszuführen, wobei die Kern-Verarbeitungsschaltung $J$ aufeinander fol-
gende Verarbeitungsstufen j umfasst, wobei jede der J aufeinander folgenden Verarbeitungsstufen j einer Zerlegungsebene der diskreten Wavelet-Transformation entspricht und mindestens einem grundlegenden Verarbeitungs-Schaltungselement (PE) auf jeder Verarbeitungsstufe implementiert ist, wobei das mindestens eine grundlegende Verarbeitungs-Schaltungselement (PE) eingerichtet ist, jeweilige Berechnungen des inneren Produkts auszuführen, die mit den k verschiedenen Filtervorgängen des diskreten Wavelet-Transformations-Vorgangs verknüpft sind, wobei die Anzahl von grundlegenden Verarbeitungs-Schaltungselementen (PE), die auf jeder der J aufeinander folgenden Verarbeitungsstufen implementiert sind, sich mit jeder höheren Zerlegungsebene um einen konstanten Faktor verringert.

2. Kern-Verarbeitungsschaltung nach Anspruch 1, wobei der konstante Faktor k ist.

3. Kern-Verarbeitungsschaltung nach Anspruch 1, wobei einer der k verschiedenen Filtervorgänge ein Tiefpass-Filtervorgang ist.


5. Kern-Verarbeitungsschaltung nach Anspruch 4, wobei der Dateneingangsblock eingerichtet ist:
   - zum parallelen Empfangen von k\(^{j}\) Eingabe-Samples eines Eingabevektors x; und
   - um die k\(^{j}\) Eingabe-Samples parallel den k\(^{j}\) parallelen Eingängen des Daten-Routing-Blocks der ersten Verarbeitungsstufe j = 1 bereitzustellen.

6. Kern-Verarbeitungsschaltung nach Anspruch 4, wobei der Dateneingangsblock eingerichtet ist:
   - zum seriellen Empfangen von k\(^{j}\) Eingabe-Samples eines Eingabevektors x; und
   - um die k\(^{j}\) Eingabe-Samples parallel den k\(^{j}\) parallelen Eingängen des Daten-Routing-Blocks der ersten Verarbeitungsstufe j = 1 bereitzustellen.


10. Kern-Verarbeitungsschaltung nach Anspruch 4, wobei der Daten-Routing-Block von Verarbeitungsstufe j eingerichtet ist zum:
   - Verketten aufeinander folgender Gruppen von k\(^{j+1}\) Ausgabewerten, die parallel von dem Tiefpass-Filtervorgang einer direkt vorhergehenden Verarbeitungsstufe j - 1 empfangen wurden, in aufeinander folgenden Arbeitsschritten, um einen Eingabevektor zu bilden, der Eingabewerte für die grundlegenden Verarbeitungs-Schaltungselemente (PE) der Verarbeitungsstufe j umfasst, wobei die Zeit, die von den grundlegenden Verarbeitungs-Schaltungselementen (PE) der Verarbeitungsstufe j - 1 benötigt wird, um eine Gruppe von k\(^{j+1}\) Ausgabewerten parallel zu erzeugen, einen Arbeitsschritt der Kern-Verarbeitungsschaltung definiert; und
- Ausgeben einer jeweiligen Gruppe von \( p \) Eingabewerten des Eingabevektors parallel an jedes grundlegende Verarbeitungs-Schaltungselement (PE) der Verarbeitungsstufe \( j \) bei nachfolgenden Zeiteinheiten, wobei eine Zeiteinheit eine Unterteilung eines Arbeitsschrittes ist, wobei die Zeiteinheit durch die Zeit definiert wird, die von den grundlegenden Verarbeitungs-Schaltungselementen (PE) benötigt wird, um einen Vorgang eines inneren Produkts der Länge \( p \) auszuführen.

11. Kern-Verarbeitungsschaltung nach Anspruch 10, wobei der Daten-Routing-Block ein Schieberegister umfasst, das eingerichtet ist, die aufeinander folgenden Gruppen von \( k^j+1 \) Ausgabewerten zu verketten, die in aufeinander folgenden Arbeitsschritten von dem Tiefpass-Filtervorgang der direkt vorhergehenden Verarbeitungsstufe \( j \cdot 1 \) empfangen wurden.

12. Kern-Verarbeitungsschaltung nach Anspruch 10, wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) jeder Verarbeitungsstufe \( j \) \( p \) Eingänge aufweist, die eingerichtet sind, \( p \) Eingabewerte parallel zu empfangen, und eingerichtet ist, um \( k \) verschiedene Filtervorgänge der Länge \( L \) auszuführen, durch Anwenden von \( k \) jeweiligen Sätzen von \( L \) Koeffizientenwerten auf einen Satz von \( L \) Eingabewerten, die in \( \lfloor L/p \rfloor \) aufeinander folgenden parallelen Gruppen an den \( p \) Eingängen während aufeinander folgenden Zeiteinheiten empfangen wurden, wobei \( r^j \) das Aufrunden zu der nächsten ganzen Zahl repräsentiert, und wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) \( k \) Ausgänge umfasst, wobei jeder Ausgang eingerichtet ist, einen Ausgabewert auszugeben, der einem Resultat von einem der \( k \) verschiedenem Filtervorgängen entspricht, während jedem Arbeitsschritt.

13. Kern-Verarbeitungsschaltung nach Anspruch 12, wobei die Filterlänge \( L \) in einem Bereich von \( 1 \) bis \( L_{\max} \) auswählbar ist.

14. Kern-Verarbeitungsschaltung nach Anspruch 10, wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) jeder Verarbeitungsstufe \( j \) \( p = L_{\max} \) Eingänge aufweist, die eingerichtet sind, \( L_{\max} \) Eingabewerte parallel zu empfangen, wobei \( L_{\max} \) einer vorbestimmten maximalen Filterlänge entspricht, und wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) eingerichtet ist, \( k \) verschiedene Filtervorgänge auszuführen, durch Anwenden von \( k \) jeweiligen Sätzen von \( L_{\max} \) Koeffizientenwerten auf einen Satz von \( L_{\max} \) Eingabewerten, die in einer gegebenen Zeiteinheit parallel an den \( L_{\max} \) Eingängen empfangen wurden, wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) \( k \) Ausgänge umfasst, wobei jeder Ausgang eingerichtet ist, während jeder Zeiteinheit einen Ausgabewert auszugeben, der einem Resultat von einem der \( k \) verschiedenen Filtervorgängen entspricht, um durch einen Arbeitsschritt bereitzustellen, der einer einzelnen Zeiteinheit entspricht.

15. Kern-Verarbeitungsschaltung nach Anspruch 14, wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) eingerichtet ist, \( k \) verschiedene Filtervorgänge der Länge \( L \) auszuführen, durch Anwenden von \( k \) jeweiligen Sätzen von \( L_{\max} \) Koeffizientenwerten auf einen Satz von \( L \) Eingabewerten, die parallel an den \( L_{\max} \) Eingängen empfangen wurden, wobei \( L < L_{\max} \) ist.


17. Kern-Verarbeitungsschaltung nach Anspruch 16, wobei das erste grundlegende Verarbeitungs-Schaltungselement (PE) eingerichtet ist, die \( p \) Zwischen-Filterwerte parallel zu empfangen.


19. Kern-Verarbeitungsschaltung nach einem der Ansprüche 1 bis 18, wobei jedes grundlegende Verarbeitungs-Schaltungselement (PE) eingerichtet ist, \( k = 2 \) verschiedene Filtervorgänge auszuführen, wobei einer der zwei verschiedenen Filtervorgänge ein Tiefpass-Filtervorgang (LP) ist, und der andere der zwei verschiedenen Filtervorgänge ein Hochpass-Filtervorgang (HP) ist.


22. Kern-Verarbeitungsschaltung nach Anspruch 10, wobei die Kern-Verarbeitungsschaltung eingerichtet ist, auf jeder Verarbeitungsstufe \( j = 2 \) bis \( J \) Sätze von \( k \) Ausgabewerten von den jeweiligen Ausgängen der grundlegenden Verarbeitungs-Schaltungselemente (PE) einer direkt vorhergehenden Verarbeitungsstufe \( j - 1 \) zu empfangen, und einen Stride-Permutations-Vorgang an einem Vektor auszuführen, der aus den Sätzen von \( k \) Ausgabewerten gebildet ist, um Ausgabewerte von ähnlichen Filtervorgängen zusammenzukuppeln, wobei die Ausgabewerte der direkt vorhergehenden Verarbeitungsstufe \( j - 1 \) dem Daten-Routing-Block der derzeitigen Verarbeitungsstufe \( j \) bereitgestellt sind.


26. Mehrkern-Verarbeitungsschaltung, umfassend eine Vielzahl von \( r \) Kern-Verarbeitungsschaltungen gemäß irgendeinem der Ansprüche 1 bis 25, wobei die Kern-Verarbeitungsschaltungen der Vielzahl \( r \) eingerichtet sind, parallel an jeweiligen Sätzen von \( k^j \) Eingabe-Samples zu operieren.


28. Verfahren zum Ausführen eines diskreten Wavelet-Transformations-Vorgangs mit einer Kern-Verarbeitungsschaltung unter Verwendung von \( k \) verschiedenen Filtervorgängen über aufeinander folgende Zerlegungsebenen, wobei die Kern-Verarbeitungsschaltung \( J \) aufeinander folgende Verarbeitungsstufen \( j \) umfasst, wobei jede der \( J \) aufeinander folgenden Verarbeitungsstufen \( j \) einer Zerlegungsebene der diskreten Wavelet-Transformation entspricht und mit mindestens einem grundlegenden Verarbeitungs-Schaltungselement (PE) auf jeder Verarbeitungsstufe implementiert ist, wobei das mindestens eine grundlegende Verarbeitungs-Schaltungselement (PE) eingerichtet ist, jeweilige Berechnungen des inneren Produkts auszuführen, die mit den \( k \) verschiedenen Filtervorgängen des diskreten Wavelet-Transformations-Vorgangs verknüpft sind, wobei das Verfahren ein Einteilen des Eingabesignal-
vektors in r aufeinander folgende Abschnitte von $k^j$ Werten und Verwenden der Kern-Verarbeitungsschaltung für jeden aufeinander folgenden Abschnitt umfasst.


Revendications

1. Circuit de traitement de noyau pour un microprocesseur, le circuit de traitement de noyau étant conçu pour effectuer une opération de transformée en ondelettes discrètes, en utilisant $k$ opérations de filtrage différentes, sur des niveaux de décomposition consécutifs, le circuit de traitement de noyau comprenant $J$ étages de traitement consécutifs $j$, chacun desdits $J$ étages de traitement consécutifs $j$ correspondant à un niveau de décomposition de la transformée en ondelettes discrètes et étant mis en oeuvre avec au moins un élément de circuit de traitement de base (PE) à chaque étage de traitement, ledit au moins un élément de circuit de traitement de base (PE) étant conçu pour effectuer des calculs de produit interne respectifs associés aux $k$ opérations de filtrage différentes de l’opération de transformée en ondelettes discrètes, le nombre d’éléments de circuit de traitement de base (PE) mis en oeuvre dans chacun desdits $J$ étages de traitement consécutifs diminuant d’un facteur constant avec chaque niveau de décomposition $j$ croissant.

2. Circuit de traitement de noyau selon la revendication 1, dans lequel ledit facteur constant est $k$.

3. Circuit de traitement de noyau selon la revendication 1, dans lequel une desdites $k$ opérations de filtrage différentes est une opération de filtrage passe-bas.

4. Circuit de traitement de noyau selon la revendication 3, dans lequel chaque étage de traitement $j = 1$ jusqu’à $J$ comprend un bloc d’acheminement de données couplé aux éléments de circuit de traitement de base respectifs (PE) de l’étage de traitement, le bloc d’acheminement des données de chaque étage de traitement respectif $j$ ayant $k^{j-1} + 1$ entrées parallèles et étant agencé pour acheminer les valeurs d’échantillonnage reçues au niveau desdites entrées parallèles vers les éléments de circuit de traitement de base (PE) de l’étage de traitement respectif, le bloc d’acheminement de données du premier étage de traitement, $j = 1$, étant conçu pour recevoir en parallèle des valeurs d’échantillonnage d’entrée $k^j$ provenant d’un bloc d’entrée de données et chacun des blocs d’acheminement de données des étages de traitement suivants, $j = 2$ jusqu’à $J$, étant conçu pour recevoir en parallèle des valeurs d’échantillonnage de sortie $k^{j-1} + 1$ issues de l’opération de filtrage passe-bas d’un étage de traitement immédiatement précédent.

5. Circuit de traitement de noyau selon la revendication 4, dans lequel le bloc d’entrée de données est conçu pour :
   - recevoir $k^j$ échantillons d’entrée d’un vecteur d’entrée $x$ en parallèle ; et
   - fournir lesdits $k^j$ échantillons d’entrée en parallèle aux $k^j$ entrées parallèles du bloc d’acheminement de données du premier étage de traitement, $j = 1$.

6. Circuit de traitement de noyau selon la revendication 4, dans lequel le bloc d’entrée de données est conçu pour :
   - recevoir $k^j$ échantillons d’entrée d’un vecteur d’entrée $x$ en série ; et
   - fournir lesdits $k^j$ échantillons d’entrée en parallèle aux $k^j$ entrées parallèles du bloc d’acheminement de données du premier étage de traitement, $j = 1$.

7. Circuit de traitement de noyau selon la revendication 4, dans lequel le bloc d’acheminement de données d’un étage
de traitement particulier est conçu pour doter l’ensemble des éléments de circuit de traitement de base (PE) de l’étage de traitement particulier avec des groupes respectifs de valeurs d’entrée p en parallèle.

8. Circuit de traitement de noyau selon la revendication 7, dans lequel les éléments de circuit de traitement de base (PE) de l’étage de traitement particulier sont conçus pour fonctionner en parallèle sur leurs groupes respectifs de valeurs d’entrée p pour effectuer les opérations de produit interne respectives de longueur p.

9. Circuit de traitement de noyau selon la revendication 8, dans lequel les éléments de circuit de traitement de base (PE) de l’étage de traitement particulier sont conçus pour fonctionner en parallèle sur des groupes consécutifs de valeurs d’entrée p reçues en parallèle afin que chacun produise, en parallèle, une valeur de sortie respective pour chacune desdites k opérations de filtrage différentes.

10. Circuit de traitement de noyau selon la revendication 4, dans lequel le bloc d’acheminement de données de l’étage de traitement j est conçu pour :

   - concaténer des groupes consécutifs de valeurs de sortie $k^{j-1} \cdot j + 1$ reçus en parallèle de l’opération de filtrage passe-bas d’un étage de traitement immédiatement précédent j - 1 dans des étapes de fonctionnement consécutifs pour former un vecteur d’entrée comprenant des valeurs d’entrée pour les éléments de circuit de traitement de base (PE) de l’étage de traitement j, la quantité de temps prise par les éléments de circuit de base (PE) de l’étage de traitement j - 1 pour produire un groupe de valeurs de sortie $k^{j-1} \cdot j + 1$ en parallèle définissant une étape de fonctionnement du circuit de traitement de noyau ; et
   - produire un groupe respectif de valeurs d’entrée p à partir dudit vecteur d’entrée vers chaque élément de circuit de traitement de base (PE) de l’étage de traitement j en parallèle au niveau d’unités de temps successives, où une unité de temps est une sous-division d’une étape de fonctionnement, ladite unité de temps étant définie par la quantité de temps prise par les éléments de circuit de traitement de base (PE) pour effectuer une opération de produit interne de longueur p.

11. Circuit de traitement de noyau selon la revendication 10, dans lequel le bloc d’acheminement de données comprend un registre à décalage conçu pour concaténer lesdits groupes consécutifs de valeurs de sortie $k^{j-1} \cdot j + 1$ reçus dans des étapes de fonctionnement consécutifs à partir de l’opération de filtrage passe-bas dudit étage de traitement immédiatement précédent j - 1.

12. Circuit de traitement de noyau selon la revendication 10, dans lequel chaque élément de circuit de traitement de base (PE) de chaque étage de traitement j possède des entrées p conçues pour recevoir des valeurs d’entrée p en parallèle et est conçu pour effectuer k opérations de filtrage différentes de longueur L en appliquant k ensembles respectifs de valeurs de coefficient L à un ensemble de valeurs d’entrée L reçues dans $\lceil L / p \rceil$ groupes parallèles consécutifs au niveau desdites entrées p au cours d’unités de temps consécutives, où $\lceil \cdot \rceil$ représente l’arrondissement au nombre entier le plus proche, et dans lequel chaque élément de circuit de traitement de base (PE) comprend k sorties, chaque sortie étant conçue pour produire une valeur de sortie correspondant à un résultat d’un dudit ensemble de k opérations de filtrage différentes au cours de chaque étape de fonctionnement.

13. Circuit de traitement de noyau selon la revendication 12, dans lequel la longueur de filtre L peut être choisie dans une plage de 1 à $L_{\text{max}}$.

14. Circuit de traitement de noyau selon la revendication 10, dans lequel chaque élément de circuit de traitement de base (PE) de chaque étage de traitement j possède des entrées p conçues pour recevoir des valeurs d’entrée $L_{\text{max}}$ en parallèle, $L_{\text{max}}$ correspondant à une longueur de filtre maximale prédéterminée, et dans lequel chaque élément de circuit de traitement de base (PE) est conçu pour effectuer k opérations de filtrage différentes en appliquant k ensembles respectifs de valeurs de coefficient $L_{\text{max}}$ à un ensemble de valeurs d’entrée $L_{\text{max}}$ reçus en parallèle au niveau desdites entrées $L_{\text{max}}$, dans une unité de temps donnée, dans lequel chaque élément de circuit de traitement de base (PE) comprend k sorties, chaque sortie étant conçue pour produire une valeur de sortie correspondant à un résultat d’une desdites k opérations de filtrage différentes au cours de chaque unité de temps, fournissant ainsi une étape de fonctionnement correspondant à une seule unité de temps.

15. Circuit de traitement de noyau selon la revendication 14, dans lequel chaque élément de circuit de traitement de base (PE) est conçu pour effectuer k opérations de filtrage différentes de longueur L en appliquant k ensembles respectifs de valeurs de coefficient L, où $L < L_{\text{max}}$, à un ensemble de valeurs d’entrée L reçues en parallèle au niveau desdites entrées $L_{\text{max}}$. 
16. Circuit de traitement de noyau selon l’une quelconque des revendications 10 à 15, dans lequel au moins un premier élément de circuit de traitement de base (PE) dans un étage de traitement j est conçu pour recevoir un ensemble de valeurs p représentant des valeurs de filtrage p intermédiaires obtenues dans un second élément de traitement de circuit de base (PE) du même étage de traitement j dans une unité de temps antérieure et ledit premier élément de circuit de traitement de base (PE) est conçu pour utiliser ledit ensemble de valeurs de filtrage p intermédiaires dans une unité de temps actuelle.

17. Circuit de traitement de noyau selon la revendication 16, dans lequel ledit premier élément de circuit de traitement de base (PE) est conçu pour recevoir lesdites valeurs p intermédiaires en parallèle.

18. Circuit de traitement de noyau selon la revendication 16, dans lequel ledit premier élément de circuit de traitement de base (PE) est conçu pour recevoir un ensemble de valeurs d’entrée p en parallèle à partir dudit second élément de circuit de traitement de base (PE) au niveau de chaque unité de temps successive, après une première unité de temps initiale.

19. Circuit de traitement de noyau selon l’une quelconque des revendications 1 à 18, dans lequel chaque élément de circuit de traitement de base (PE) est conçu pour effectuer k = 2 opérations de filtrage différentes, dans lequel une des deux opérations de filtrage différentes est une opération de filtrage passe-bas (LP) et l’autre desdites deux opérations de filtrage différentes est une opération de filtrage passe-haut (HP).

20. Circuit de traitement de noyau selon la revendication 10, dans lequel chaque élément de circuit de traitement de base (PE) de chaque étage de traitement j possède des entrées p conçues pour recevoir des valeurs d’entrée p en parallèle et est conçu pour effectuer deux opérations de filtrage différentes de longueur L en appliquant deux ensembles respectifs de valeurs de coefficient L à un ensemble de valeurs d’entrée L reçues dans [L / p] groupes parallèles consécutifs au niveau desdites p entrées au cours d’unités de temps consécutives, où [*] représente l’arrondissement au nombre entier le plus proche, une première desdites deux opérations de filtrage différentes étant une opération de filtrage passe-bas (LP) et une seconde desdites deux opérations de filtrage différentes étant une opération de filtrage passe-haut (HP), et dans lequel chaque élément de circuit de traitement de base (PE) comprend deux sorties, une première sortie conçue pour produire une valeur de sortie filtrée passe-bas correspondant à un résultat de l’opération de filtrage passe-bas (LP) effectuée sur ledit ensemble de valeurs d’entrée L au cours de chaque étape de fonctionnement, et une seconde sortie étant conçue pour produire une valeur de sortie filtrée passe-haut correspondant à un résultat de l’opération de filtrage passe-haut (HP) effectuée sur ledit ensemble de valeurs d’entrée L au cours de chaque étape de fonctionnement.

21. Circuit de traitement de noyau selon la revendication 20, dans lequel le circuit de traitement de noyau est conçu pour recevoir, à chaque étage de traitement j = 2 jusqu’à J, des paires de valeurs de sortie des sorties respectives des éléments de circuit de traitement de base (PE) d’un étage de traitement immédiatement précédent j - 1 et effectuer une opération de tri parfaite sur un vecteur formé à partir desdites paires de valeurs de sortie pour grouper ensemble les valeurs de sortie provenant d’opérations de filtrage similaires, lesdites valeurs de sortie provenant de l’étage de traitement immédiatement précédent j - 1 devant être fournies en tant que valeurs d’entrées au bloc d’acheminement de données de l’étage de traitement j actuel.

22. Circuit de traitement de noyau selon la revendication 10, dans lequel le circuit de traitement de noyau est conçu pour recevoir, à chaque étage de traitement j = 2 jusqu’à J, des ensembles de valeurs de sortie k des sorties respectives des éléments de circuit de traitement de base (PE) d’un étage de traitement immédiatement précédent j - 1, et pour effectuer une opération de permutation de pas sur un vecteur formé à partir desdits ensembles de k valeurs de sortie pour grouper ensemble les valeurs de sortie issues d’opérations de filtrage similaires, lesdites valeurs de sortie issues de l’étage de traitement immédiatement précédent j - 1 devant être fournies au bloc d’acheminement de données de l’étage de traitement j actuel.

23. Circuit de traitement de noyau selon la revendication 19, dans lequel le circuit de traitement de noyau est conçu de manière à ce qu’une transformée en ondelettes discrètes aboutisse à un niveau de décomposition j en formant un vecteur à partir des valeurs de sortie d’une opération de filtrage passe-haut et d’une opération de filtrage passe-bas à l’étage de traitement j et des valeurs de sortie issues des opérations de filtrage passe-haut de tous les étages de traitement précédents.

24. Circuit de traitement de noyau selon l’une quelconque des revendications 1 à 23, intégré dans un circuit de microprocesseur avec un second circuit de traitement pour effectuer une transformée en ondelettes discrètes, ledit second
circuit de traitement étant conçu pour recevoir la sortie du circuit de traitement de noyau et effectuer au moins un autre niveau de décomposition.

25. Circuit de traitement de noyau selon la revendication 24, dans lequel ledit second circuit de traitement est conçu pour effectuer un algorithme pyramidal récursif.

26. Circuit de traitement multinoyau comprenant une pluralité \( r \) de circuits de traitement de noyau selon l’une quelconque des revendications 1 à 25, les circuits de traitement de noyau de ladite pluralité \( r \) étant conçus pour fonctionner en parallèle sur des ensembles respectifs de \( k^j \) échantillons d’entrée.

27. Microprocesseur comprenant un circuit de traitement de noyau selon l’une quelconque des revendications 1 à 25 ou une pluralité \( r \) de circuits de traitement de noyau selon la revendication 26.

28. Procédé de réalisation d’une opération de transformée en ondelettes discrètes avec un circuit de traitement de noyau utilisant \( k \) opérations de filtrage différentes, sur des niveaux de décomposition consécutifs, le circuit de traitement de noyau comprenant \( J \) étages de traitement consécutifs \( j \), chacun desdits \( J \) étages de traitement consécutifs \( j \) correspondant à un niveau de décomposition de la transformée en ondelettes discrètes et étant mis en œuvre avec au moins un élément de circuit de traitement de base (PE) à chaque étage de traitement, ledit au moins un élément de circuit de traitement de base (PE) étant conçu pour effectuer des calculs de produit interne respectifs associés aux \( k \) opérations de filtrage différentes de l’opération de transformée en ondelettes discrètes, le procédé comprenant la mise en portion du vecteur de signal d’entrée en \( r \) parties consécutives de valeurs \( k^j \) et à utiliser le circuit de traitement de noyau pour chaque partie consécutive.

29. Procédé de réalisation d’une opération de transformée en ondelettes discrètes avec un circuit de traitement multinoyau, utilisant \( k \) opérations de filtrage différentes, sur des niveaux de décomposition consécutifs, le circuit de traitement multinoyau comprenant une pluralité \( r \) de circuits de traitement de noyau, chaque circuit de traitement de noyau comprenant \( J \) étages de traitement consécutifs \( j \), chacun desdits \( J \) étages de traitement consécutifs \( j \) correspondant à un niveau de décomposition de la transformée en ondelettes discrètes et étant mis en œuvre avec au moins un élément de circuit de traitement de base (PE) dans chaque circuit de traitement de noyau à chaque étage de traitement, chacun desdits éléments de circuit de traitement de base (PE) étant conçu pour effectuer les calculs de produit interne respectifs associés aux \( k \) opérations de filtrage différentes de l’opération de transformée en ondelettes discrètes, le procédé consistant à portionner le vecteur de signal d’entrée en parties consécutives de valeurs \( r \times k^j \) et à utiliser ladite pluralité \( r \) de circuits de traitement de noyau en parallèle sur des parties consécutives de valeurs \( r \times k^j \).
Figure 1. Tree-structured representation of a 1-D discrete wavelet transform.
\( \tilde{x}^{(0)} = x \)

\( \tilde{x}^{(1)} = D_1 \cdot \tilde{x}^{(0)} \)

\( \tilde{x}^{(2)} = D_2 \cdot \tilde{x}^{(1)} \)

\( \tilde{x}^{(3)} = D_3 \cdot \tilde{x}^{(2)} = y \)

*Figure 2a*

*Figure 2* Flowgraph representation of a 1-D discrete wavelet transform \((N=16, L=4, J=3)\)
Figure 4a and 4b. The general structure of Type 1 and Type 2 core DWT architectures
Figure 5. A realization of the $j$th pipeline stage, $j=1,\ldots,J$, of the Type 1 core DWT architecture
Figure 6. An example of realization of the Type 1 core DWT architecture for the case

\[ p = L_{\text{max}} = 6; \quad J = 3; \quad N = 2^m, \quad m = 3,4,... \]
Figure 7. An example of realization of the Type 1 core DWT Architecture for the case $L_{\text{max}}=6; J_{\text{max}}=3; p=2; N=2^m, m=3,4,...$
Figure 8a. Possible structures for the Pes used in Type 1 core DWT architecture: arbitrary p
Figure 8b. Possible structures for the PEs used in Type 1 core DWT architecture; p=1
Figure 8c. Possible structures for the PEs used in Type 1 core DWT architecture: p=2

A similar part for the high-pass filter
Figure 8d. Possible structures for the PEs used in Type 1 core DWT architecture: $p = l_{\text{max}}$. A tree of adders.

Input 1

Input 2

Input $l_{\text{max}}$

Output 1

Output 2
Figure 9. A realization of the $j$th pipeline stage, $i=1,...,J$, of the Type 2 core DWT architecture.
Figure 10a. An example of realization of the Type 2 core DWT architecture for the case $L_{max}=6; J=3; p=2; N=2^m, m=3,4,...$: (a) the general structure
Figure 10b. An example of realization of the Type 2 core DWT architecture for the case $L_{\text{max}}=6$; $J=3$; $p=2$; $N=2^m$, $m=3,4,\ldots$: (b) a possible structure for the PEs
Figure 11a and 11b. The general structure of multi-core DWT architectures.
Figure 11a and 11b. The general structure of multi-core DWT architectures.

Data routing block at stage J

From Fig. 11a.

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A core DWT architecture with $J_{\text{min}}$ pipeline stages and $p$ basic units within every PE

\[ p \geq \left\lceil \frac{K}{2} \right\rceil \]

Figure 12a. The variable resolution DWT architecture: (a) based on a single-core DWT architecture
Figure 12b. The variable resolution DWT architecture: (b) based on a multi-core DWT architecture.
Figure 13. Delay versus number of basic units for some known and proposed DWT architectures (DWT parameters are: $N=1024, J=3, L=9$).
Table 1. Comparative performance of some DWT architectures

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Area, $A$</th>
<th>Period, $T_p$</th>
<th>$AT_p^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMS320C64x [36]</td>
<td>$2(64$-bit)</td>
<td>$4N(1-2^{-J})+25J$</td>
<td>$32(1-2^{-J})N+6J^2$</td>
</tr>
<tr>
<td>Architectures in [14], [15]</td>
<td>$L$</td>
<td>$2N$</td>
<td>$4N^2L$</td>
</tr>
<tr>
<td>Architectures in [14]–[19]</td>
<td>$2L$</td>
<td>$N$</td>
<td>$2N^2L$</td>
</tr>
<tr>
<td>Architectures in [12], [24]</td>
<td>$JL$</td>
<td>$N$</td>
<td>$JN^2L$</td>
</tr>
<tr>
<td>Architectures of [30]</td>
<td>$\sum_{j=1}^{4L} \left[ L/2^{J-2} \right]$</td>
<td>$N/2$</td>
<td>$N^2L$</td>
</tr>
<tr>
<td>FPP DWT [34] (pipelined)</td>
<td>$2NL(1-1/2^J)$</td>
<td>$1$ (per vector)</td>
<td>$2NL(1-1/2^J)$</td>
</tr>
<tr>
<td>LPP DWT [34]</td>
<td>$2L(2^J-1)$</td>
<td>$N/2^J$</td>
<td>$N^2L(2^J-1)/2^{2J-1}$</td>
</tr>
<tr>
<td>Single-core DWT (Type 1 or 2)</td>
<td>$2p(2^J-1)$</td>
<td>$NL/p$</td>
<td>$N^2p(L/p)^2/2^{2J-1}$</td>
</tr>
<tr>
<td>Single-core DWT, $p=1$</td>
<td>$2(2^J-1)$</td>
<td>$NL/2^J$</td>
<td>$N^2L/2^{J-1}$</td>
</tr>
<tr>
<td>Single-core DWT, $p=L_{max}$ ($L \leq L_{max}$)</td>
<td>$2L_{max}(2^J-1)$</td>
<td>$N/2^J$</td>
<td>$N^2L_{max}/2^{J-1}$</td>
</tr>
<tr>
<td>Multi-core DWT</td>
<td>$2p(2^J-1)$</td>
<td>$(NL/p)/(2^{J+1})$</td>
<td>$(N^2p(L/p)^2)/(2^{J+1})$</td>
</tr>
<tr>
<td>Multi-core DWT, $r=4$, $p=1$</td>
<td>$8(2^J-1)$</td>
<td>$NL/2^{J+2}$</td>
<td>$N^2L/2^{J+1}$</td>
</tr>
<tr>
<td>Multi-core DWT, $r=4$, $p=L_{max}$ ($L \leq L_{max}$)</td>
<td>$2L_{max}(2^J-1)$</td>
<td>$N/(2^{J+1})$</td>
<td>$(N^2L_{max})/(2^{J+1})$</td>
</tr>
<tr>
<td>Variable resolution single-core DWT</td>
<td>$2p(2^{J_{min}}-1)+K$</td>
<td>$K2^J_{min}$</td>
<td>$N[2L/K]2^{J_{min}}$</td>
</tr>
<tr>
<td>$p \geq [K/2]$ ($K \leq 2L$)</td>
<td>$K2^J_{min}$</td>
<td>$=2NL/(K2^{J_{min}})$</td>
<td>$K2^{J_{min}}$</td>
</tr>
</tbody>
</table>
Figure 15.
Table 2. Numerical examples from Table 1 ($N=1024, L=9, J=3$ and $J=4$)

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Number of BUs (gate count)</th>
<th>Period, $T_p$ (in cc's)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J=3</td>
<td>J=4</td>
</tr>
<tr>
<td>TMS320C64x</td>
<td>2 64-bit</td>
<td>2 64-bit</td>
</tr>
<tr>
<td>Non-pipelined, [14], [15]</td>
<td>9 (17226)</td>
<td>9 (17226)</td>
</tr>
<tr>
<td>Two-stage pipelined, [14]-[19]</td>
<td>18 (34452)</td>
<td>18 (34452)</td>
</tr>
<tr>
<td>J-stage pipelined, [12], [24]</td>
<td>27 (51678)</td>
<td>36 (68904)</td>
</tr>
<tr>
<td>J-stage pipelined, [30]</td>
<td>36 (68904)</td>
<td>36 (68904)</td>
</tr>
<tr>
<td>FPP DWT (pipelined), [34]</td>
<td>16128 (3.08 $\cdot 10^5$)</td>
<td>17280 (3.3 $\cdot 10^5$)</td>
</tr>
<tr>
<td>LPP DWT (pipelined), [34]</td>
<td>126 (241164)</td>
<td>270 (516780)</td>
</tr>
<tr>
<td>Single-core DWT, $p=1$</td>
<td>14 (26796)</td>
<td>30 (57420)</td>
</tr>
<tr>
<td>Single-core DWT, $p=L_{max}=10$</td>
<td>140 (267960)</td>
<td>300 (574200)</td>
</tr>
<tr>
<td>Multi-core DWT, $r=4$, $p=1$</td>
<td>56 (107184)</td>
<td>120 (229680)</td>
</tr>
<tr>
<td>Multi-core DWT, $r=4$, $p=L_{max}=10$</td>
<td>560 (107184)</td>
<td>1200 (229680)</td>
</tr>
<tr>
<td>Variable resolution single-core DWT, $p=1$, $(K=2)$, $J_{min}=2$</td>
<td>8 (15312)</td>
<td>8 (15312)</td>
</tr>
<tr>
<td>Variable resolution single-core DWT, $p=2$, $(K=3/K=4)$, $J_{min}=2$</td>
<td>15 (28710)</td>
<td>15 (28710)</td>
</tr>
<tr>
<td>Variable resolution single-core DWT, $p=3$, $(K=6)$, $J_{min}=2$</td>
<td>16 (30624)</td>
<td>16 (30624)</td>
</tr>
<tr>
<td>Variable resolution single-core DWT, $p=3$, $(K=6)$, $J_{min}=2$</td>
<td>24 (45936)</td>
<td>24 (45936)</td>
</tr>
<tr>
<td>Variable resolution single-core DWT, $p=5$, $(K=9)$, $J_{min}=2$</td>
<td>39 (74646)</td>
<td>39 (74646)</td>
</tr>
</tbody>
</table>
Figure 16.
Table 3. Numerical examples from Table 1 (\(N=1024\), \(L=5\), \(J=3\) and \(J=4\))

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Number of BUs (gate count)</th>
<th>Period, (T_p) (in cc's)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(J=3)</td>
<td>(J=4)</td>
</tr>
<tr>
<td>TMS320C64x</td>
<td>2 64-bit</td>
<td>2 64-bit</td>
</tr>
<tr>
<td>Non-pipelined, [14], [15]</td>
<td>5 (9570)</td>
<td>5 (9570)</td>
</tr>
<tr>
<td>Two-stage pipelined, [14]-[19]</td>
<td>10 (19140)</td>
<td>10 (19140)</td>
</tr>
<tr>
<td>J-stage pipelined, [12], [24]</td>
<td>15 (28710)</td>
<td>20 (38280)</td>
</tr>
<tr>
<td></td>
<td>20 (38280) or</td>
<td>20 (38280) or</td>
</tr>
<tr>
<td>J-stage pipelined, [30]</td>
<td>15 (28710) or</td>
<td>18 (34452)</td>
</tr>
<tr>
<td>FPP DWT (pipelined), [34]</td>
<td>8960 (1.7 (\times) (10^8))</td>
<td>9600 (1.8 (\times) (10^8))</td>
</tr>
<tr>
<td>LPP DWT (pipelined), [34]</td>
<td>70 (133980)</td>
<td>150 (287100)</td>
</tr>
<tr>
<td>Single-core DWT, (p=1)</td>
<td>14 (26796)</td>
<td>30 (57420)</td>
</tr>
<tr>
<td>Single-core DWT, (p=L_{max}=5)</td>
<td>70 (133980)</td>
<td>150 (287100)</td>
</tr>
<tr>
<td>Multi-core DWT, (r=4), (p=1)</td>
<td>56 (107184)</td>
<td>120 (229680)</td>
</tr>
<tr>
<td>Multi-core DWT,(r=4,p=L_{max}=5)</td>
<td>280 (535920)</td>
<td>600 (1148400)</td>
</tr>
<tr>
<td>Variable resolution single-core DWT, (p=1), (K=2), (J_{min}=2)</td>
<td>8 (15312)</td>
<td>8 (15312)</td>
</tr>
</tbody>
</table>

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REFERENCES CITED IN THE DESCRIPTION

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