Convention Application for a Patent

31071/77

INTERNATIONAL BUSINESS MACHINES CORPORATION

of Armonk, New York, 10504, United States of America

hereby apply for the grant of a Patent for an invention entitled

"ADAPTIVE PHASE DETECTION METHOD AND APPARATUS"

which is described in the accompanying complete specification. This application is a Convention Application and is based on the application numbered 76-39689 for a patent or similar protection made in France on 23rd December, 1976.

My address for service is:

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Dated this FIRST day of NOVEMBER 1977

INTERNATIONAL BUSINESS MACHINES CORPORATION

By: ________________________________
Signature of Applicant

Registered Patent Attorney

To: The Commissioner of Patents
DECLARATION IN SUPPORT OF CONVENTION OR NON-CONVENTION APPLICATION FOR A PATENT OR PATENT OF ADDITION

(The declaration shall be made by the applicant, or, if the applicant is a body corporate, by a person authorized by the body corporate to make the declaration on its behalf).

In support of the Application made for a patent 

"ADAPTIVE PHASE DETECTION METHOD AND APPARATUS"

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I do solemnly and sincerely declare as follows:-

I am authorized by:

International Business Machines Corporation

the applicant for the patent to make this declaration on its behalf.

Applicant is entitled to apply by virtue of an assignment dated August 24, 1977 by Andrzej Tadeusz Milewski and August 24, 1977 by Dominique Noel Godard to IBM France, and IBM France assigned their rights to the invention to the applicant INTERNATIONAL BUSINESS MACHINES CORPORATION.

Applicant is entitled to make the application as follows:-

The basic application as defined by Section 141 of the Act was made in France on the 23rd December 1976

by IBM France

The basic application referred to in paragraph 3 of this Declaration was the first application made in a Convention country in respect of the invention the subject of the application.

Declared at Don Mills this 11th day of November, 1977

Signature(s) of declarant(s).

(No attestation or other signature is required).

Note: Initial all Alterations.
Phase detection apparatus is also claimed.

CLAIM

1. In a data transmission system using the FSK modulation technique wherein the phase of the signal transmitted at each signaling instant may be any one of the phases of a M-phase constellation, a phase detection method supplying an estimated value \( \hat{\phi}_n \) of the phase transmitted at signaling instant \( nT \) and compensating for the effects of the intersymbol interference created by not more than one leading lobe and/or an arbitrary number \( N \) of trailing lobes of the impulse response of the transmission channel characterized in that it comprises the steps of:

   a) determining the phase \( \phi_n \) of the signal received at signaling instant \( nT \),

   b) determining at least two residual errors, each of which is obtained by subtracting from received phase \( \phi_n \) one of the phases of the constellation
and an estimated value of the phase error created by the intersymbol interference and corresponding to that phase of the constellation,
c) comparing the residual errors thus obtained with each other and selecting the smallest,
d) selecting as phase $\hat{\phi}_n$ that phase of the constellation which yields the smallest residual error, and
e) selectively adjusting the estimated phase errors in accordance with the residual errors.
Complete Specification for the invention entitled:

"Adaptive Phase Detection Method and Apparatus"

The following statement is a full description of this invention, including the best method of performing it known to me:
This invention relates generally to systems designed to compensate for the linear distortions introduced by the transmission channels of digital data transmission systems. More particularly, the invention relates to an adaptive phase detection method and apparatus for a digital data transmission system that uses the phase-shift keying (PSK) modulation technique.

The widely used PSK modulation technique is described, for example, in the books entitled "Data Transmission", by R.W. Bennett and J.R. Davey, Chapter 10, McGraw-Hill, New York, 1965, and "Principles of Data Communication", by R.W. Lucky, J. Salz and E.J. Weldon, Jr., Chapter 3, McGraw-Hill, New York, 1968. In the PSK modulation technique, the sequence of bits to be transmitted is first converted into a sequence of symbols each of which can take on a discrete number of values that is generally equal to a power of two. These symbols are then transmitted one at a time, at instants which have a T-second spacing and are called signaling instants, each symbol taking the form of a pulse modulated by a carrier whose phase exhibits a given change relative to the phase of the immediately preceding symbol.

The modulated pulses are fed to a transmission channel whose output is connected to a data receiver. The function of the transmission channel is to provide a signal relatively similar to the input signal applied thereto. The receiver examines the signal received from the transmission channel at each signaling instant and determines its phase and the transmitted data. In actual practice, mainly for reasons of cost, the telephone lines of the public network are most commonly used as transmission channels. However, telephone lines, while satisfactory for voice transmission purposes, become inadequate if used to transmit data pulses at a relatively high rate with a very low probability of error. On any telephone line of a given quality, there will appear beyond a certain pulse transmission rate amplitude and phase distortions that will alter the shape of the pulses being transmitted.
These distortions will create at the signaling instants an interaction between successive pulses, making it difficult for the receiver to correctly detect the data. This interaction is known as intersymbol interference. To compensate for the effects of the intersymbol interference, the receiver is provided with a device called an equalizer.

The most widely used type of equalizer is the so-called automatic adaptive equalizer which has been discussed in many publications and is described, for example, in Chapter 6 of the book by R.W. Lucky and al. mentioned earlier. An equalizer generally consists of a network whose transfer function is adjusted to meet a given performance criterion. Such an equalizer is a complicated device which requires a very high computing power and is, consequently, very expensive. Its incorporation in a receiver will, therefore, considerably increase the cost of the latter. The decision as to whether an equalizer should be incorporated in a receiver will of course depend on the performance level required to enable the receiver to operate satisfactorily, but also on the cost advantage to be gained thereby. An equalizer must be used in certain cases, for example, in the event of the transmission rate being equal to or higher than 4800 bits per second (bps). On the other hand, an equalizer will not be required if, for example, the transmission rate is lower than 2400 bps or if the quality of the line to be used is very high. However, the use of an equalizer would be unnecessary but desirable in some cases, for example in a receiver designed to accommodate a great many different lines some of which might have marginal characteristics in relation to accepted standards. In these latter cases, it is apparent that there is a need for a device capable of compensating for the effects of the intersymbol interference without however requiring the very high computing power associated with an equalizer. Accordingly, the present invention has sought to improve
the phase detection system by realizing an apparatus that would compensate for the effects of the intersymbol interference.

It is therefore an object of the present invention to provide an adaptive phase detection method and apparatus which compensates for the effects of the intersymbol interference.

It is another object of the invention to provide an adaptive phase detection method and apparatus which only requires a low computing power compared with that required by an equalizer.

It is still another object of the invention to provide an adaptive phase detection method and apparatus which can readily be integrated using existing technologies.

These and other objects are generally achieved through the use of an adaptive phase detection method and apparatus which provides an estimated value \( \hat{\phi}_n \) of the phase \( \phi_n \) transmitted at signaling instant \( nT \) in a data transmission system using a M-phase PSK modulation technique, and compensates for the effects of the intersymbol interference created by not more than one leading lobe and an arbitrary number of trailing lobes of the impulse response of the transmission channel. The method of the present invention mainly comprises the steps of:

- determining at least two residual errors, each of which is obtained by subtracting from the phase \( \gamma_n \) received at signaling instant \( nT \) one of the possible values which can be taken on by the signal transmitted at each signaling instant, i.e. one of the phases of the constellation, and an estimated value of the phase error due to the intersymbol interference and corresponding to that phase of the constellation,

- comparing the residual errors thus obtained with each other,
deciding that the phase $\hat{\phi}_n$ with the maximum likelihood of being phase $\phi_n$ is that phase of the constellation which yields the smallest of the residual errors, and selectively adjusting the estimated phase errors in accordance with the residual errors.

The foregoing and other objects, features and advantages of the invention will be apparent from the following more particular description of a preferred embodiment of the invention, as illustrated in the accompanying drawings.

Figures 1A-1D represent waveforms helpful in gaining a better understanding of the invention.

Figure 2 is a block diagram of a PSK receiver incorporating the phase detector of the invention.

Figure 3, consisting of Figures 3A and 3B, represents an embodiment of the invention.

Figure 3C represents an exemplary embodiment of address selection circuit 42 of Figure 3.

Figure 4, consisting of Figures 4A and 4B, illustrates another embodiment of the invention.

Figure 4C illustrates an exemplary embodiment of address selection circuit 113' of Figure 4.

Illustrates an apparatus which, when added to the apparatus of Figure 4, represents another embodiment of the invention.

In order that the present invention may be more readily understood, it is believed necessary to first describe the problems which existed and are solved by the invention. To this end, the intersymbol interference phenomenon will be described with reference to Figures 1A-1D. Figure 1A generally represents the shape of an isolated data pulse $S_o$ supplied by the transmitter of a data transmission system using the PSK modulation technique. Such a pulse is often referred to as a signal element and is described, for example, in Chapter 4 of the book by FR9-76-011.
R.W. Lucky et al mentioned earlier. The shape of this pulse is selected so as to approximate as closely as possible the impulse response of the transmission channel. The pulse shown in Figure 1A corresponds to the transmission of an isolated phase value at signaling instant $t_0$. In the illustrated example, the pulse extends over six T-second signaling periods and includes a main lobe centered at $t_0$, two leading lobes, and two trailing lobes. The phase of the pulse at $t_0$ is representative of the phase being transmitted. It should be noted that although the pulse extends over six signaling periods, its amplitude is nonzero at $t_0$ and zero at the adjacent signaling instants $t_{-3}, t_{-2}, t_{-1}, t_1, t_2, t_3$. Thus, another phase value can be transmitted at instant $t_1$ by sending a pulse $S_1$ similar to $S_0$ but centered at $t_1$. It will be appreciated that pulses $S_0$ and $S_1$ do not interfere with each other at the signaling instants and that, assuming the use of an ideal transmission channel introducing no noise or distortion, an examination of the signal received at instants $t_0$ and $t_1$ would permit retrieving the exact phases transmitted. In practice, the transmission channel would introduce amplitude and phase distortions, so that the pulses obtained at the receiving end would be distorted.

Figure 1B represents the isolated pulse $S_0$ as distorted by the transmission channel and received at the input of the receiver. The distorted pulse $S_0$ is designated $S'_0$ in figure 1B. Note that the amplitude of pulse $S'_0$ is no longer zero at instants $t_{-3}, t_{-2}, t_{-1}, t_1, t_2$ and $t_3$. Figure 1C represents the isolated pulse $S_1$ as distorted by the transmission channel, this pulse being designated $S'_1$. Figure 1D illustrates a distorted, isolated pulse $S_2$ centered at instant $t_2$ and designated $S'_2$. While the three pulses $S'_0, S'_1, S'_2$ have been isolated in figures 1B, 1C and 1D, respectively, it will be appreciated that these pulses are, in fact, superimposed and that as
a result the signal fed to the receiver is a composite signal. It will be observed that at each signaling instant there is an interference between pulses $S'_0$, $S'_1$ and $S'_2$. For example, at instant $t_0$, there is an interference between the main lobe of $S'_0$, the first trailing lobe of $S'_1$ and the second trailing lobe of $S'_2$. This interference modifies the amplitude of the signal transmitted at each signaling instant and may result in the data being incorrectly detected by the receiver.

Figures 1A-1D illustrate a pulse, or impulse response of the transmission channel, that includes two leading lobes and two trailing lobes; in practice, the number of these lobes, also called secondary lobes, can vary and only some of them contribute significantly to the intersymbol interference. It is clear from figures 1B-1D that the interference modifies the amplitude of the transmitted pulses, but it will readily be apparent without having to resort to a mathematical discussion that it also modifies the phase of these pulses at the signaling instants.

As has been mentioned, it is the object of this invention to provide a phase detection method and apparatus for minimizing the effects of intersymbol interference in a system that uses the PSK modulation technique. To illustrate the context within which the invention finds application, a simplified block diagram of a PSK receiver incorporating the phase detector of the present invention has been shown in figure 2. For the sake of simplicity, only those parts of the receiver which are necessary to implement the invention have been shown in the figure.

The signal received from the transmission channel is applied via line 1 to the input of a sampling device 2 which samples the signal at a rate of $K/T$ Hz, $K$ being an integer, where $1/T$ Hz represents the signaling rate. Device 2 supplies the values assumed by the signal received at instants which are $nT$-multiples of the signaling instants, or samples of the received signal. These samples are fed to an analog-to-digital converter 3 which converts them to digital values. The digital samples are then fed to a Hilbert transformer 4 which provides on lines 5 and 6 the in-phase and quadrature components, respectively, of the signal.
received at the signaling instants. The Hilbert transformer 4 is a well-known device that is widely used in data transmission. This transformer is, in fact, a digital filter whose transfer function is

\[ H(f) = \exp(-j\pi/2) \times \text{sign of } f \]

The in-phase and quadrature components supplied by the Hilbert transformer 4 are applied via lines 5 and 6 to the input of a resolver 7 which provides the digitally coded values of the phase of the signal received at the signaling instants. A description of a resolver is given, for example, in an article entitled "The Cordic Trigonometric Computing Technique", by J.E. Valder, in IRE Transactions on Electronic Computers, pp. 330-334, September 1959. Another type of resolver is described in French Patent No. 71 47850 filed by the present applicant December 21, 1971, and entitled "Déetecteur de phase digital" (Digital Phase Detector), publication No. 2 164 544. The phase values supplied by resolver 7 are applied via line 8 to the input of the phase detector 9 of the present invention which derives the transmitted data therefrom. The phase values are also fed via line 10 to a clock recovery device 11 which derives therefrom a clock signal defining the sampling instants. Many types of clock recovery system are currently available; a description of such a device may be found, for example, in French Patent application No. 76 21564 filed by the present applicant July 9, 1976 and entitled "Procédé et dispositif de synchronisation de l'horloge du récepteur d'un système de transmission de données en modulation PSK" (Method and Device for Synchronizing the Receiver Clock in a Data Transmission System Using PSK Modulation). The clock signal supplied by clock recovery device 11 controls sampling device 2.

The phase detection method and apparatus of the present invention will now be described in detail.

In a data transmission system which uses the PSK modulation technique, the phase \( \gamma_n \) of the signal received at instant \( nT \), or \( n^{th} \) signaling instant, may be expressed as

\[ \gamma_n = \phi_n + \epsilon_n \]

where

\[ \phi_n = \sum_{k=0}^{n-1} h(k) \]

\[ \epsilon_n = \sum_{k=n}^{\infty} h(k) \]

and

\[ h(k) = \text{rect}(k/T) \]

The rect function is defined as

\[ \text{rect}(t) = \begin{cases} 1 & \text{for } |t| < 0.5 \\ 0 & \text{otherwise} \end{cases} \]
\( \phi_n \) is the phase of the signal transmitted at instant \( nT \), and 
\( \varepsilon_n \) is the phase error due to intersymbol interference which alters the received phase \( \gamma_n \).

In expression (1), the effects of other noise factors, compensation for which is outside the scope of this invention, have been ignored.

The method of the present invention permits minimizing the effects of the intersymbol interference created by the first leading lobe and/or an arbitrary number of trailing lobes of the impulse response of the transmission channel. In order that the invention may be more readily understood, detailed descriptions will now be given of three successive cases wherein the interference is created by the first trailing lobe, this being termed Case A hereafter, then by the first trailing lobe and the first leading lobe (Case B), and finally by the first leading lobe and the first two trailing lobes (Case C). On the basis of these three cases, those skilled in the art should experience no difficulty in extending the inventive method to an arbitrary number of trailing lobes.

**Case A – First Trailing Lobe**

This case assumes that the transmission channel is such that only the first trailing lobe of the impulse response will create significant intersymbol interference.

Phase \( \gamma_n \) will be altered by an interference term created by the first trailing lobe of the pulse corresponding to the phase \( \phi_{n-1} \) transmitted at instant \( (n-1)T \). The phase error \( \varepsilon_n \) is only dependent upon the phase change \( (\phi_{n-1} - \phi_n) \). For clarity, such a phase error will be written

\[ \varepsilon_n = (\phi_{n-1} - \phi_n) \]

In a transmission system using a M-phase PSK modulation technique, the phase transmitted at each signaling instant can assume a value...
selected from a finite number of \( M \) distinct values. Such a finite set of values is often referred to as a "constellation". The \( M \) phases of the constellation will be written

\[
\phi^t \quad \text{for} \quad t = 0, 1, 2, \ldots, (M-1)
\]

with, for example,

\[
(2) \quad \phi^t = \frac{2\pi}{M} t = 0, 1, 2, \ldots, (M-1).
\]

The phase change \((\phi^t_{n-1} - \phi^t_n)\) can also take on \( M \) distinct values:

\[
(3) \quad \phi^t_{n-1} - \phi^t_n = m \frac{2\pi}{M}; \quad m = 0, 1, 2, \ldots, (M-1).
\]

The phase error \( \xi_n (\phi^t_{n-1} - \phi^t_n) \) can also take on \( M \) possible distinct values:

\[
\xi_n (m \frac{2\pi}{M}) \quad m = 0, 1, \ldots, (M-1).
\]

In accordance with the detection method of the present invention, it is assumed that at instant \( nT \) an estimated value \( \hat{\phi}^t_{n-1} \) of the phase \( \phi^t_{n-1} \) and estimates \( \hat{\xi}_n \) \((m \frac{2\pi}{M})\) of the \( M \) possible values of the phase error are available. Accordingly, the present method comprises the following steps:

**Step 1**

This consists in calculating the \( M \) residual errors

\[
(4) \quad \xi_n^t = \gamma_n - \phi^t - \hat{\xi}_n (\hat{\phi}^t_{n-1} - \phi^t) \quad \text{for} \quad t = 0, 1, \ldots, (M-1)
\]

where

\[
\hat{\xi}_n (\hat{\phi}^t_{n-1} - \phi^t) \quad \text{is an estimated value of} \quad \xi_n (\hat{\phi}^t_{n-1} - \phi^t)
\]

Note that the estimates of the possible values of the phase error, or estimated phase errors,

\[
\hat{\xi}_n (\hat{\phi}^t_{n-1} - \phi^t), \quad t = 0, 1, \ldots, (M-1)
\]

designate the estimated phase errors

\[
\hat{\xi}_n (m \frac{2\pi}{M}), \quad m = 0, 1, \ldots, (M-1).
\]

The notation \( \hat{\xi}_n (\hat{\phi}^t_{n-1} - \phi^t) \) enables each value of \( \phi^t \) to be associated with the corresponding phase error in expression \((4)\).

In what follows, an estimated value of a possible phase error will be referred to as an estimated phase error.
Step 2

The phase $\hat{\phi}_n$ with the maximum likelihood of being the phase $\phi_n$ transmitted at time $nT$ is that phase $\phi^i$ of the constellation which yields the smallest residual error

$$\min_{i} |\gamma_n - |\gamma_n^i| = \hat{\phi}_n = \phi^i$$

Step 3

Obviously, the $M$ estimated phase errors are not known a priori. According to the invention, they are determined in an adaptive manner. At the end of step 2, it was assumed that phase $\hat{\phi}_n$ was phase $\phi^i$ of the constellation. A new estimate of one of the possible values of the phase error which will be used at instant $(n+1)T$ is obtained in accordance with the expression

$$\gamma_n = \gamma_n - \phi^i = \gamma_n - \phi = \gamma_n - \gamma_n^i = \gamma_n + \phi^i = \gamma_n + \phi - \phi^i = \gamma_n + \phi^i = \gamma_n^i$$

where

$\gamma$ is a small positive constant which may be, for example, a negative power of 2.

Note that a single estimated phase error is adjusted at each signaling instant. For example, if

$$\hat{\phi}_{n-1} - \phi^i = \frac{2\pi}{M}$$

at instant $(n+1)T$, then the following estimated values will be used:

$$\gamma_{n+1}^m = \gamma_n^m + \frac{2\pi}{M}$$

for $m \neq m_i$

and

$$\gamma_{n+1} = \gamma_n^m + \frac{2\pi}{M}$$

as defined by (6).
Note Step 1 of the above process theoretically requires the calculation of \( M \) residual errors \( \varepsilon_n \) defined by relation (4). In practice, by first determining which two phases of the constellation are closest to \( y_n \), the calculation of the \( M \) residual errors can be limited to those which involve said two phases, as is done in the apparatus illustrated in figure 3, which will be described later.

**Case B – First Leading Lobe and First Trailing Lobe**

The method of the present invention will now be described in relation to the case wherein the first leading lobe and the first trailing lobe of the impulse response of the transmission channel create significant intersymbol interference. In this case, phase \( y_n \) is altered by an interference term created by the first trailing lobe of the pulse corresponding to phase \( \phi_{n-1} \) and by the first leading lobe of the pulse corresponding to phase \( \phi_{n+1} \) transmitted at instant \((n+1)T\).

The phase error which alters the received phase \( y_n \) is dependent upon the phase changes \((\phi_{n+1} - \phi_n)\) and \((\phi_{n-1} - \phi_n)\) which, for example, will be

\[
\phi_{n+1} - \phi_n = \frac{2\pi}{M} \text{ for } j = 0, 1, \ldots, (M-1)
\]

\[
\phi_{n-1} - \phi_n = k\frac{2\pi}{M} \text{ for } k = 0, 1, \ldots, (M-1)
\]

The phase error \( \varepsilon_n \) will be written

\[
\varepsilon_n = 0 (\phi_{n+1} - \phi_n, \phi_{n-1} - \phi_n)
\]
Since each of the phase changes $\phi_{n+1} - \phi_n$ and $\phi_{n-1} - \phi_n$ can take on $M$ distinct values, the phase error will assume $M^2$ distinct values.

To determine $\hat{\phi}_n$, it will be assumed that received phases $\psi_{n+1}$ and $\psi_n$ are available as well as the estimated value $\hat{\phi}_{n-1}$ of phase $\phi_{n-1}$ and the estimated values of the $M^2$ phase errors. Observe that the availability of $\psi_{n+1}$ implies that the value of phase $\hat{\phi}_n$ is determined at instant $(n+1)T$. The method of the invention comprises the following steps:

1. **Step 1**

   Since phase $\phi_{n+1}$ is unknown, the $M^2$ residual errors must be calculated:

   \[
   \xi_n = \psi_n - \phi_n = \hat{\phi}_n + (\phi^j - \phi_0, \phi_{n-1} - \phi^l)
   \]

   for $j, l = 0, 1, \ldots, (M-1)$

   where $\phi^j$ represents the $M$ phases of the constellation that $\phi_{n+1}$ can assume.

2. **Step 2**

   The phase $\hat{\phi}_n$ with the maximum likelihood of being phase $\phi_n$ is that phase of the constellation which yields the smallest residual error.

   \[
   \min_{j, l} | \xi^j_n | = \min_{j, l} | \xi^j_p | \Rightarrow \hat{\phi}_n = \phi^p
   \]

3. **Step 3**

   As in step 3 of Case A, the estimated phase errors are determined in an adaptive manner.
The direct application to Case B of the adaptive adjustment method of Case A, as defined by expression (6), would yield the expression

\[ \hat{\phi}_{n+1} (\phi - \hat{\phi}, \phi - \hat{\phi}, \phi - \hat{\phi}) = \hat{\phi}_{n} (\phi - \hat{\phi}, \phi - \hat{\phi}, \phi - \hat{\phi}) + \gamma^2 \epsilon_n \]

where

\[ \epsilon_n^2 \text{ is the smallest residual error.} \]

Actually, this expression is dependent upon of the phase change \( \phi_{n+1} - \hat{\phi}_n \) which is indeterminate since \( \phi_{n+1} \) is unknown. It is therefore preferable in Case B to define the adaptive adjustment method by means of expression (9):

\[ (9) \quad \hat{\phi}_{n+1} (\phi - \hat{\phi}, \phi - \hat{\phi}, \phi - \hat{\phi}) = \hat{\phi}_{n} (\phi - \hat{\phi}, \phi - \hat{\phi}, \phi - \hat{\phi}) + \gamma^2 \epsilon_{n-1} \]

where

\[ \epsilon_{n-1} \text{ is the residual error that was observed at the preceding signaling instant and corresponds to the combination of phase changes} \]

\[ \hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_n \]

and

\[ \gamma^2 \text{ is a small positive constant which may be equal to a negative power of 2.} \]

Note that, as in Case A, only one of the \( N^2 \) estimated phase errors is adjusted at each signaling instant, the other estimated values remaining unchanged.
Note

As in Case A, the number of residual errors to be calculated in step 1 can be reduced by identifying the two phases of the constellation which are closest to $\psi_{n+1}$ and $\psi_n$ and by calculating only those residual errors which involve said two phases, as is done in the apparatus of figure 4 to be described later.

Case C - First Leading Lobe and First Two Trailing Lobes

Where intersymbol interference is created by the first leading lobe and the first two trailing lobes of the impulse response of the transmission channel, phase $\psi_n$ is altered by the first leading lobe of the pulse corresponding to $\phi_{n+1}$, by the first trailing lobe of the pulse corresponding to $\phi_{n-1}$, and by the second trailing lobe of the pulse corresponding to $\phi_{n-2}$.

The phase error is dependent upon the phase changes $$(\phi_{n+1} - \phi_n), (\phi_{n-1} - \phi_n),$$ and $$(\phi_{n-2} - \phi_n)$$

For example, the following will be obtained:

$$\phi_{n+1} - \phi_n = j \frac{2\pi}{M} \quad j = 0, 1, \ldots, (M-1)$$

$$\phi_{n-1} - \phi_n = k \frac{2\pi}{M} \quad k = 0, 1, \ldots, (M-1)$$

$$\phi_{n-2} - \phi_n = s \frac{2\pi}{M} \quad s = 0, 1, \ldots, (M-1)$$

Since each of these phase changes can assume $M$ distinct values, the phase error can take on $M^3$ distinct values.
However, if the second trailing lobe is relatively small compared with the first leading lobe and the first trailing lobe (which would be true in practice), a good approximation to the phase error can be written as:

\[
\epsilon_n = \hat{\phi}_n (\phi_n - \phi_{n+1} - \phi_{n-1} - \phi_{n-2}) + \hat{\chi}_n (\phi_n - \phi_{n-1})
\]

The phase error appears in expression (10) as a sum of two phase error terms.

To obtain \(\hat{\phi}_n\), it will be assumed that received phases \(\psi_{n+1}\) and \(\psi_n\) are available as well as estimated phases \(\hat{\phi}_{n-1}\), \(\hat{\phi}_{n-2}\) and estimated values of the two terms of the possible phase errors:

\[
\hat{\phi}_n (\phi_n - \phi_{n+1} - \phi_{n-1} - \phi_{n-2}) \quad \text{and} \quad \hat{\chi}_n (\phi_n - \phi_{n-1})
\]

In this instance, the method of the present invention includes the following steps:

**Step 1**

This consists in calculating the residual errors:

\[
\epsilon_{j+l} = \psi_{n-j+l} \hat{\phi}_n (\phi_n - \phi_{n+1} - \phi_{n-1} - \phi_{n-2}) - \hat{\chi}_n (\phi_n - \phi_{n-1})
\]

for \(j, l = 0, 1, \ldots, (M-1)\).

Note that we have:

\[
\epsilon_{j+l} = \epsilon_{n} + \hat{\chi}_n (\phi_n - \phi_{n-1})
\]

where \(\epsilon_{j+l}\) is the residual error obtained in step 1 of Case B, but only represents a partial residual error in Case C.
Step 2

The phase $\hat{\phi}_n$ with the maximum likelihood of being phase $\phi_n$ is that phase of the constellation which yields the smallest of the residual errors $\xi_n$.

Step 3

The two estimated phase error terms $\hat{\phi}_n$ and $\hat{\chi}_n$ are determined separately in an adaptive manner.

The term $\hat{\phi}_n$ is adjusted as in step 3 of Case B, except that $\hat{\epsilon}_{n-1}$ is substituted for $\xi_{n-1}$.

Expression (9) becomes:

$$(9') \hat{\phi}_{n+1} = \hat{\phi}_n - \hat{\phi}_{n-1} - \hat{\phi}_{n-2} - \hat{\phi}_{n-3}$$

where

$\hat{\epsilon}_{n-1}$ is the residual error observed at the preceding signaling instant and which corresponds to the combination of phase changes $\hat{\phi}_n$, $\hat{\phi}_{n-1}$, $\hat{\phi}_{n-2}$, and $\hat{\phi}_{n-3}$, and

$\gamma$ is a small positive constant which may be equal to a negative power of 2.

The term $\hat{\chi}_n$ is adjusted as follows:

To ensure that the adjustment of term $\hat{\chi}_n$ and that of term $\hat{\phi}_n$ are coherent as per (9'), a new value of $\hat{\chi}_n$ is obtained in accordance with expression (12) below

$$(12) \quad \hat{\chi}_{n+1} = \hat{\chi}_n + \gamma \hat{\epsilon}_{n-1}$$

where

$\hat{\epsilon}_{n-1}$ is the residual error observed at the preceding signaling instant and which corresponds to the combination of phase changes $\hat{\phi}_n$, $\hat{\phi}_{n-1}$, $\hat{\phi}_{n-2}$, and $\hat{\phi}_{n-3}$.
Only one value of each of terms $\hat{\theta}_n$ and $\hat{\chi}_n$ is adjusted at each signaling instant, the other estimated values remaining unchanged.

**Note**

What has been stated above in the Notes on Cases A and B also applies to Case C and to the apparatus illustrated in figure 5, which will be described later.

Those skilled in the art will appreciate that the phase detection method just described in relation to Cases A-C and to be complemented by a description of the devices shown in figures 3-5, can also be used to compensate for the effects of the intersymbol interference created by not more than the first leading lobe and/or an arbitrary number of trailing lobes.

The method of the invention, in its preferred, most general form, may be defined as follows: (To prevent confusion with Cases A and B, some notations have been modified)

Where the intersymbol interference is created solely by N trailing lobes, the inventive method mainly comprises the steps of:

- calculating at least two of the residual errors

\[ \hat{E}_n = \gamma_n - \varphi_n - \hat{E}_n (\hat{\varphi}_{n-1} - \varphi_n, \hat{\varphi}_{n-2} - \varphi_n, \ldots, \hat{\varphi}_{n-N} - \varphi_n) \]

where

- $\varphi_n$ represents the phases of the constellation,

and

- $\hat{E}_n$ represents the estimated phase errors,
comparing the residual errors so obtained with each other,

- selecting as phase \( \hat{\phi}_n \) that phase of the constellation which yields the smallest residual error, and

- adjusting the estimated phase error corresponding to phase \( \hat{\phi}_n \) in accordance with the smallest residual error.

Where the intersymbol interference is created by the first leading lobe and \( N \) trailing lobes, the inventive method mainly comprises the steps of:

- calculating at least two of the residual errors

\[
E_n^J = \gamma_n^J - \phi_n^J - E_n^L (\phi_n^L, \phi_{n-1}^L, \phi_{n-2}^L, \ldots, \phi_{n-N}^L - \phi_n^L)
\]

where

\( \phi_n^J, \phi_n^L \) represent the phases of the constellation,

- comparing the residual errors so obtained with each other,

- selecting as phase \( \hat{\phi}_n \) that phase of the constellation which yields the smallest residual error, and

- adjusting the estimated phase error corresponding to the combination of phase changes

\[
\hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-1}, \ldots, \hat{\phi}_n^{(N+1)} - \hat{\phi}_{n-1}
\]

in accordance with the residual error observed at the preceding signaling instant and which corresponds to that combination of phase changes.
Exemplary embodiments of the phase detection apparatus of the present invention will now be described with reference to figure 3 to 5.

**FIGURE 3 - CASE "A"**

An exemplary embodiment of the phase detector of the invention which is used to implement the inventive method in Case A, with identification of the two phases of the constellation which are closest to the received phase, will first be described. The transmission system employed will be assumed to use a 4-phase PSK modulation technique.

The phase transmitted at each signaling instant can assume one of the four phases of the constellation which will be written

$$\phi^l \quad \text{for} \quad l = 0, 1, 2, 3$$

with, for example,

$$\phi^0 = 0, \pi/2, \pi, \text{ and } 3\pi/2.$$  

The phases received and transmitted at instant nT will be designated $\psi_n$ and $\phi_n$, respectively.

The phase change ($\phi_{n-1} - \phi_n$) can assume one of four distinct values

$$\phi_{n-1} - \phi_n = 0, \pi/2, \pi, \text{ and } 3\pi/2.$$  

The phase error can also assume the distinct values

$$\epsilon_n (\phi_{n-1} - \phi^l) \quad \text{for} \quad l = 0, 1, 2, 3.$$  

For clarity, these may be written

$$\epsilon_n (0), \epsilon_n (\pi/2), \epsilon_n (\pi), \text{ and } \epsilon_n (3\pi/2).$$  

It is assumed that at signaling instant nT the estimated value $\hat{\phi}_{n-1}$ is available along with estimated values of the
possible phase errors, or estimated phase errors

\[ \hat{\varepsilon}_n, \hat{\varepsilon}_n^{(\pi/2)}, \hat{\varepsilon}_n^{(\pi)}, \text{ and } \hat{\varepsilon}_n^{(3\pi/2)} \]

The method of the present invention, as implemented in the apparatus of figure 3, is as follows:

**Step 1**

To directly perform the previously described step 1 of Case A, it is necessary to calculate the four residual errors

\[ \varepsilon_n^l = \gamma_n - \psi_n - \hat{\varepsilon}_n (\hat{\varphi}_n - \psi_k) \text{ for } l = 0, 1, 2, 3. \]

In practice, this first step may be split into two sub-steps as follows:

**Step 1-1**

This consists in selecting those phases of the constellation which are closest to received phases \( \gamma_n \), said two phases being designated \( \varphi^1 \) and \( \varphi^2 \).

**Step 1-2**

This consists in calculating the two residual errors

\[ \varepsilon_n^{(1)} = \gamma_n - \psi_n - \hat{\varepsilon}_n (\hat{\varphi}_{n-1} - \psi^1) \]

\[ \varepsilon_n^{(2)} = \gamma_n - \psi_n - \hat{\varepsilon}_n (\hat{\varphi}_{n-1} - \psi^2) \]

**Step 2**

This consists in selecting as phase \( \hat{\varphi}_n \) whichever of phases \( \varphi^1 \) and \( \varphi^2 \) yields the smallest of the two residual errors so calculated.
Step 3

A new estimated value of one of the four possible phase errors is obtained in accordance with expression (6) which may be written:

\[ \hat{\xi}_{n+1} (\hat{\phi}_{n-1} - \hat{\phi}) = \hat{\xi}_n (\hat{\phi}_{n-1} - \hat{\phi}) + \gamma \xi_n \]

where

\[ \xi_n = \xi_{n-1} \text{ if } \hat{\phi}_n = \phi^1 \]

\[ \xi_n = \xi_{n-1} \text{ if } \hat{\phi}_n = \phi^2 \]

The other estimated values of the possible phase errors remain unchanged.

For example, if \( \hat{\phi}_n = \phi^1 = 0 \) and \( \hat{\phi}_{n-1} = \pi/2 \), we get

\[ \hat{\xi}_{n+1} (\pi/2) = \xi_n (\pi/2) + \gamma \xi_n \]

\[ \hat{\xi}_{n+1} (\pi) = \xi_n (\pi) \]

\[ \hat{\xi}_{n+1} (3\pi/2) = \xi_n (3\pi/2) \]

\[ \hat{\xi}_{n+1} (0) = \xi_n (0) \]

The estimated values \( \hat{\xi}_{n+1} (0), \hat{\xi}_n (\pi/2), \hat{\xi}_n (\pi) \) and \( \hat{\xi}_{n+1} (3\pi/2) \) will be used at the next signaling instant to detect \( \hat{\phi}_{n+1} \).
The apparatus shown in figure 3 which uses the above method will now be described.

Detection of $\psi_n$ (Steps 1 and 2)

The received phase $\psi_n$, as supplied by the resolver 7 (figure 2), is fed via line 8 to a selection logic 20 which provides on lines 21 and 22 the phases $\phi^1$ and $\phi^2$ of the constellation which are closest to $\psi_n$. In this example, the constellation comprises four phases, namely, 0, $\pi/2$, $\pi$ and $3\pi/2$, to which are associated the following couples of bits, or dibits:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>01</td>
</tr>
<tr>
<td>$\pi$</td>
<td>10</td>
</tr>
<tr>
<td>$3\pi/2$</td>
<td>11</td>
</tr>
</tbody>
</table>

Selection logic 20 provides phases $\phi^1$ and $\phi^2$ by determining the signs of quantities $\psi_n-\pi/2$, $\psi_n-\pi$, and $\psi_n-3\pi/2$.

To this end, phase $\psi_n$ present on line 8 is applied in parallel to the (+) inputs of three binary adders AD1, AD2 and AD3 whose (-) inputs receive binary coded quantities $\pi/2$, $\pi$ and $3\pi/2$, respectively. The outputs of adders AD1, AD2 and AD3 are respectively connected to the inputs of three sign detectors 23, 24 and 25 which respectively supply the sign of quantities $\psi_n-\pi/2$, $\psi_n-\pi$ and $\psi_n-3\pi/2$. Sign detectors 23-25 supply a 0 bit if they receive a negative input and a 1 bit if the input is positive. The outputs of adders AD1-AD3 are added up by a two-bit binary adder AD4.
whose output provides the two bits representing the coded value of $\phi^1$ on line 21. Phase $\phi^2$ is obtained by means of a modulo-4 addition of 1 to the coded value of $\phi^1$ in the two-bit adder AD5. The output of AD5 is connected to line 22. Selection logic 20 performs step 1-1 of the inventive method.

Step 1-2 of the inventive method requires that the residual errors $\xi_n^1$ and $\xi_n^2$ defined by expression (14) and (15) be calculated. To this end, the estimated phase errors $\xi_n(\hat{\phi}_{n-1} - \phi^1)$ and $\xi_n(\hat{\phi}_{n-1} - \phi^2)$ must first be determined. The phase changes to be considered in order to determine phase errors $\xi_n(\hat{\phi}_{n-1} - \phi^1)$ and $\xi_n(\hat{\phi}_{n-1} - \phi^2)$ are $\phi_{n-1} - \phi^1$ and $\phi_{n-1} - \phi^2$. To calculate these, phase $\phi^1$ is applied via line 21 to the (-) input of a two-bit binary adder AD6 whose (+) input receives via line 26 the two bits representing the coded value of phase $\hat{\phi}_{n-1}$. Phase change $\phi_{n-1} - \phi^1$ is supplied by added AD6 on line 27. Similarly, phase $\phi^2$ is applied via line 22 to the (-) input of a two-bit binary adder AD7 whose (+) input receives phase $\hat{\phi}_{n-1}$ via line 26. Phase change $\hat{\phi}_{n-1} - \phi^2$ is supplied by AD7 on line 28. Coded phase changes $\hat{\phi}_{n-1} - \phi^1$ and $\hat{\phi}_{n-1} - \phi^2$ represent the addresses of the two estimated phase errors $\xi_n(\hat{\phi}_{n-1} - \phi^1)$ and $\xi_n(\hat{\phi}_{n-1} - \phi^2)$ stored in a 4-position random-access memory (RAM) 29. Phase changes $\hat{\phi}_{n-1} - \phi^1$ and $\hat{\phi}_{n-1} - \phi^2$ available on lines 27 and 28 are fed to an addressing circuit 30 which controls the addressing of RAM 29. It will be appreciated by those skilled
in the art that circuit 30 will sequentially address the two addresses available on lines 27 and 28. The estimated phase errors $\hat{\epsilon}_n (\phi_{n-1} - \phi^1)$ and $\hat{\epsilon}_n (\phi_{n-1} - \phi^2)$ read out of memory 29 are available on lines 31 and 32. For clarity, the output register of memory 29 has not been shown in the figure, but those skilled in the art will readily understand that the estimated phase errors are successively read out of the memory and stored in such a register and will then be simultaneously available on lines 31 and 32. Expressions (14) and (15) require that quantities $\gamma_n - \phi^1$ and $\gamma_n - \phi^2$ be calculated. To this end, phases $\phi^1$ and $\phi^2$ available in coded form on lines 21 and 22 are converted into radians by multiplying same by $\pi/2$. Phase $\phi^1$ is applied via line 33 to one of the two inputs of a multiplier 34 the other input of which receives the quantity $\gamma/2$. Phase $\phi^2$ is applied via line 35 to one of the two inputs of a multiplier 36 the other input of which receives the quantity of $\gamma/2$. Note that since phases $\phi^1$ and $\phi^2$ are defined by two couples of bits, the multiplications can be replaced by two additions or by a table look-up operation. The quantity $\gamma_n - \phi^1$ is calculated by adder AD8 whose (-) input is connected to the output of multiplier 34 and whose (+) input receives phase $\gamma_n$ via line 37. The quantity $\gamma_n - \phi^2$ is calculated by adder AD9 whose (-) input is connected to the output of multiplier 36 and whose (+) input receives phase $\gamma_n$ via line 37. The quantity $\gamma_n - \phi^1$ available at the output of adder AD8 is applied via line 38 to the (+) input of an adder AD10 whose (-) input receives via line 31 the estimated value $\hat{\epsilon}_n (\phi_{n-1} - \phi^1)$ read out of
memory 29. Adder AD10 supplies on line 39 the residual error $\varepsilon_n$ defined by expression (14). The quantity $\gamma_n - \phi^2$ available at the output of adder AD9 is applied via line 40 to the (+) input of an adder AD11 whose (-) input receives via line 32 the estimated value $\hat{\varepsilon}_n (\hat{\phi}_{n-1} - \phi^2)$ read out of memory 29. Adder AD11 supplies on line 41 the residual error $\varepsilon_n^2$ defined by expression (15). Adders AD6-AD11, addressing circuit 30 and memory 29 serve to perform step 1-2 of the inventive method.

Step 2 of the inventive method involves comparing residual errors $\varepsilon_n^1$ and $\varepsilon_n^2$ and selecting whichever of phases $\phi^1$ and $\phi^2$ yields the smallest of these two residual errors. The latter errors are fed via lines 39 and 41 to an address selection circuit 42 which selects the address corresponding to the residual error with the smallest absolute value. An exemplary embodiment of address selection circuit 42 is shown in figure 3C, which will be described later. This address, which is the phase change $\hat{\phi}_{n-1} - \hat{\phi}_n$, is available on the output line 43 of selection circuit 42. The phase change $\hat{\phi}_{n-1} - \hat{\phi}_n$ present on line 43 is compared with phase changes $\hat{\phi}_{n-1} - \phi^1$ and $\hat{\phi}_{n-1} - \phi^2$, respectively available on lines 27 and 28, in the comparison and selection circuit 44 which supplies phase $\hat{\phi}_n$. The value of this phase is

$$\hat{\phi}_n = \phi^1 \text{ if } \hat{\phi}_{n-1} - \hat{\phi}_n = \phi_{n-1} - \phi^1$$

and

$$\hat{\phi}_n = \phi^2 \text{ if } \hat{\phi}_{n-1} - \hat{\phi}_n = \phi_{n-1} - \phi^2$$

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In circuit 44, phase change $\phi_{n-1} - \phi_n$ is applied in parallel via line 43 to a first input of two comparators COMP 1 and COMP 2 to the second inputs of which phase changes $\phi_{n-1} - \phi^1$ and $\phi_{n-1} - \phi^2$ are respectively applied via lines 45 and 16.

Comparators COMP 1 and COMP 2 generate a 1 bit when the phases being compared are equal. The outputs of these comparators are respectively applied to a first input of two AND gates 47 and 48 to the second inputs of which phases $\phi^1$ and $\phi^2$ are respectively applied via lines 49 and 50. The outputs of AND gates 47 and 48 are applied to the inputs of an OR gate 51 which supplies a 2-bit output representing the coded value of phase $\hat{\phi}_n$. The output of OR gate 51 is connected via line 52 to the data output line of the detection apparatus of the present invention. Phase $\hat{\phi}_n$ is additionally applied via line 52 to the input of a delay element 53 which introduces a T-second delay and provides phase $\hat{\phi}_{n-1}$ on line 26.

**Adaptive Determination (Step 3)**

The adaptive manner in which the estimated phase errors are determined in accordance with expression (16) will now be described. Quantities $\hat{\phi}_{n-1} - \phi^1$, $\hat{\phi}_{n-1} - \phi^2$, $\xi_n$ and $\eta_n$ are respectively stored in delay elements 54, 55, 56 and 57 during the period when the detection of $\hat{\phi}_n$ is taking place.

The estimated phase error to be adjusted is that which corresponds to phase change $\hat{\phi}_{n-1} - \hat{\phi}_n$. The address of this phase change, which is in fact its value, is calculated by adder AD12 to whose (-) and (+) inputs are respectively applied phase $\hat{\phi}_n$ via line 58 and phase $\hat{\phi}_{n-1}$ obtained by...
delaying phase $\hat{\phi}_n$ by means of delay element 59. The phase change $\hat{\phi}_{n-1} - \hat{\phi}_n$ supplied by AD12 on output line 60 is compared with phase changes $\hat{\phi}_{n-1} - \phi^1$ and $\hat{\phi}_{n-1} - \phi^2$ respectively available on the output lines 61 and 62 of delay elements 54 and 55. Phase change $\hat{\phi}_{n-1} - \hat{\phi}_n$ is compared with $\phi_{n-1} - \phi^1$ in comparator COMP3 and with $\phi_{n-1} - \phi^2$ in comparator COMP4. Comparators COMP3 and COMP4 are identical with comparators COMP1 and COMP2. The outputs of COMP3 and delay element 56 are fed to an AND gate 63, while the outputs of COMP4 and delay element 57 are fed to an AND gate 64. The outputs of AND gates 63 and 64 are in turn fed to an OR gate 65. If $\hat{\phi}_{n-1} - \hat{\phi}_n = \hat{\phi}_{n-1} - \phi^1$ (COMP3), residual error $\xi^1_n$ will be obtained at the output of OR 65; if $\hat{\phi}_{n-1} - \hat{\phi}_n = \hat{\phi}_{n-1} - \phi^2$ (COMP4), residual error $\xi^2_n$ will be obtained. Phase change $\hat{\phi}_{n-1} - \hat{\phi}_n$ is additionally applied via line 66 to the addressing circuit 30 of memory 29, thereby causing the estimated phase error $\xi_n (\hat{\phi}_{n-1} - \hat{\phi}_n)$ to be read out of the memory. This estimated phase error is available on line 67 and applied to a (+) input of an adder AD13. The residual error $\xi^1_n$ available at the output of OR gate 65 is multiplied by the constant $\gamma$ in multiplier 68 and applied to the other (+) input of AD13. In this example, a value of $\gamma = 1/32$ has been selected and the multiplication has been replaced with a shift of four bits to the right. An updated value of the estimated phase error, $\hat{\xi}_{n+1} (\hat{\phi}_{n-1} - \hat{\phi}_n)$, is obtained at the output of added AD13. This new value is entered into memory 29 through input register 69 and stored at the address $\hat{\phi}_{n-1} - \hat{\phi}_n$ under control of the read/write line R/W. The detection of phase $\hat{\phi}_{n+1}$ can then take place at instant $(n+1)T$. 

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An exemplary embodiment of address selection circuit 42 will now be described with reference to figure 3C. The residual errors $\xi_{n}$ and $\eta_{n}$ respectively available on output lines 39 and 41 of adders AD10 and AD11 are respectively fed to circuits 70 and 71 which supply the absolute values thereof. These circuits simply consist of registers in which the signal bit is masked. The absolute value $|\omega_{2}|$ supplied by circuit 71 is subtracted by adder AD14 from the absolute value $|\omega_{1}|$ supplied by circuit 70, and the sign of the difference is detected by a sign detector 72 which generates a 0 bit if the difference is negative and a 1 bit if it is positive. The output of sign detector 72 is directly applied to an input of an AND gate 73 and through an inverter 74 to an input of an AND gate 75. The phase change $\hat{\phi}_{n-1} - \phi_{2}$ available at the output of adder AD7 is applied to the other input of AND gate 73 and the phase change $\hat{\phi}_{n-1} - \phi_{1}$ available at the output of adder AD6 is applied to the other input of AND gate 75. The outputs of AND gates 73 and 75 are inputted to an OR gate 76 whose output provides on line 43 the address corresponding to the smallest of the residual errors $\omega_{1}$ and $\omega_{2}$.

**FIGURE 4 - CASE "B"**

Figure 4 illustrates an exemplary embodiment of the phase detector of the present invention which serves to implement the inventive method as used in Case B, with identification of the two phases of the constellation which are closest to received phases $\psi_{n}$ and $\psi_{n+1}$. The transmission system employed will be assumed to use a 4-phase PSK modulation technique with a constellation.
for \( l = 0, 1, 2, 3 \)

with, for example

\[
\phi^l = 0, \pi/2, \pi, 3\pi/2
\]

Phase changes \( \phi_{n+1} - \phi_n \) and \( \phi_{n-1} - \phi_n \) can assume the following values:

\[
\phi_{n+1} - \phi_n = 0, \pi/2, \pi, 3\pi/2
\]

\[
\phi_{n-1} - \phi_n = 0, \pi/2, \pi, 3\pi/2
\]

The phase error \( \xi_n \) which alters received phase \( \psi_n \) is dependent upon phase changes \( \phi_{n+1} - \phi_n \) and \( \phi_{n-1} - \phi_n \).

Phase error \( \xi_n \) will be written

\[
\xi_n = \theta_n (\phi_{n+1} - \phi_n, \phi_{n-1} - \phi_n)
\]

Since phase changes \( \phi_{n+1} - \phi_n \) and \( \phi_{n-1} - \phi_n \) can take on four distinct values each, the phase error can assume sixteen distinct values.

In contradiction to Case A, the detection of phase \( \hat{\phi}_n \) takes place at instant \((n+1)T\) in Case B since it is necessary that received phases \( \psi_n \) and \( \psi_{n+1} \) be known. It will be assumed that the estimated value \( \hat{\phi}_{n-1} \) and the sixteen estimated values \( \hat{\theta}_n (\phi_{n+1} - \phi_{n-1} - \phi_n) \) are available.

The method of the present invention, as implemented in the apparatus of figure 4, is as follows:

**Step 1**

To directly perform the previously described step 1 of Case A, it is necessary to calculate the sixteen residual errors

\[
\tilde{r}_n = \psi_n - \phi^j - \hat{\theta}_n (\phi^j - \phi^l, \phi_{n-1} - \phi^l)
\]

for \( j, l = 0, 1, 2, 3 \)

where

\( \phi^j \) also represents the four phases of the
constellation.

In practice, this step may be split into two sub-steps as follows:

**Step 1-1**

This consists in selecting the two phases of the constellation which are closest to received phase $\psi_n$, these two phases being designated $\phi_1$ and $\phi_2$, and the two phases of the constellation which are closest to phase $\psi_{n+1}$, these latter phases being designated $\phi_3$ and $\phi_4$.

**Step 1-2**

This consists in calculating the four residual errors

\begin{align*}
\varepsilon_3^1 &= \psi_n - \phi_1 - \hat{\theta}_n (\phi_3 - \phi_1, \phi_{n-1} - \phi_1) \\
\varepsilon_3^2 &= \psi_n - \phi_2 - \hat{\theta}_n (\phi_3 - \phi_2, \phi_{n-1} - \phi_2) \\
\varepsilon_4^1 &= \psi_n - \phi_1 - \hat{\theta}_n (\phi_4 - \phi_1, \phi_{n-1} - \phi_1) \\
\varepsilon_4^2 &= \psi_n - \phi_2 - \hat{\theta}_n (\phi_4 - \phi_2, \phi_{n-1} - \phi_2)
\end{align*}

**Step 2**

If the smallest residual error is $\varepsilon_3^1$ or $\varepsilon_4^1$, then

\[ \hat{\phi}_n = \phi_1. \]

If the smallest residual error is $\varepsilon_3^2$ or $\varepsilon_4^2$, then

\[ \hat{\phi}_n = \phi_2. \]

**Step 3**

A new estimated value of the phase error is obtained in accordance with relation (22):

\[ \frac{1}{2} \hat{\theta}_{n+1} (\hat{\phi}_n, \hat{\phi}_{n-1}, \hat{\phi}_{n-2}, \hat{\phi}_{n-3}) = \hat{\theta}_n (\phi_n, \phi_{n-1}, \phi_{n-2}, \phi_{n-3}) + \sum_{k=1}^{n} \varepsilon_{k-1} \hat{\varepsilon}_{n-1} \]

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where $\xi_{n-1}$ is the residual error observed at the preceding signaling instant and which corresponds to the combination of phase changes $\phi_n - \phi_{n-1}'$, $\phi_{n-2}' - \phi_{n-1}'$

The other estimated phase errors remain unchanged.

The apparatus shown in figure 4 which is used to implement the above method will now be described.

Detection of $\hat{\phi}_n$ (Steps 1 and 2)

As has been mentioned, the detection of $\hat{\phi}_n$ takes place at the instant $(n+1)T$ when phase $\psi_{n+1}$ is received. Received phase $\psi_{n+1}$ is inputted via line 8 to a delay element 80 which introduces a delay of $T$ seconds. The phase $\psi_n$ available at the output of delay element 80 is inputted to a selection logic 20' which is similar to selection logic 20 and figure 3 and supplies on lines 81 and 82 the phase $\phi_1$ and $\phi_2$ of the constellation that are closest to phase $\psi_n$. Phase $\psi_{n+1}$ present on line 8 is also inputted via line 83 to a selection logic 20'' which is identical with logics 20 and 20' and provides on lines 84 and 85 the phase $\phi_3$ and $\phi_4$ of the constellation that are closest to phase $\psi_{n+1}$. Phases $\phi_1$, $\phi_2$, $\phi_3$ and $\phi_4$ are coded using the same code as in Case A. Selection logics 20' and 20'' serve to perform step 1-1 of the inventive method.

Step 1-2 of the inventive method calls for the calculation of the residual errors $\xi_{n+1}$, $\xi_{n+2}$, $\xi_{n+4}$ and $\xi_{n+4}$ defined by relations (18) to (21).
The six phase changes
\[ \phi^3 - \phi^1, \phi^4 - \phi^1, \phi^3 - \phi^2, \phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^1 \text{ and } \hat{\phi}_{n-1} - \phi^2 \]
must first be determined. Each of these phase changes is represented by two bits and is readily obtained by means of a modulo-4 calculation of the phase differences, this being carried out by the two-bit adders AD15-AD20. Phase \( \phi^3 \) present on line 84 is applied to the (+) input of adder AD15 while phase \( \phi^1 \) is applied to the (-) input thereof via lines 81 and 86. Phase change \( \phi^3 - \phi^1 \) is obtained on output line 87 of AD15. Phase \( \phi^3 \) present on line 84 is also applied to the (+) input of adder AD16 while phase \( \phi^2 \) is applied to the (-) input thereof via lines 82 and 88. Phase change \( \phi^3 - \phi^2 \) is provided on output line 89 of AD16. Phase \( \phi^4 \) present on line 85 is applied to the (+) input of adder AD17 while phase \( \phi^1 \) is applied to the (-) input thereof via lines 81 and 86. Phase change \( \phi^4 - \phi^1 \) is obtained on output line 90 of AD17. Phase \( \phi^4 \) is also applied to the (+) input of adder AD18 while phase \( \phi^2 \) is applied to the (-) input thereof via line 88. Phase change \( \phi^4 - \phi^2 \) is available on output line 91 of AD18. Phase \( \phi^1 \) present on line 81 is applied to the (-) input of adder AD19 whose (+) input receives phase \( \hat{\phi}_{n-1} \) available on line 92. Phase change \( \hat{\phi}_{n-1} - \phi^1 \) is available on output line 93 of AD19. Phase \( \phi^2 \) present on line 82 is applied to the (-) input of AD20 while phase \( \hat{\phi}_{n-1} \) is applied to the (+) input thereof via line 92. Phase change \( \hat{\phi}_{n-1} - \phi^2 \) is available on output line 94 of AD20. The outputs of adders AD15-AD18 are multiplied by four (shift of two bits to the left) in multipliers 95-98, respectively. The
outputs of multipliers 95 and 97 and the output of adder AD19 are concatenated by OR circuits 99 and 101, respectively. The outputs of multipliers 96 and 98 and the output of adder AD20 concatenated chained by OR circuits 100 and 102, respectively. There are thus obtained at the outputs of the four OR circuits 99-102 four 4-bit words designated A31, A32, A41 and A42 that respectively correspond to the following combinations of phase changes.

A31 : $\phi^3 - \phi^1, \phi_{n-1} - \phi^1$
A32 : $\phi^3 - \phi^2, \phi_{n-1} - \phi^2$
A41 : $\phi^4 - \phi^1, \phi_{n-1} - \phi^1$
A42 : $\phi^4 - \phi^2, \phi_{n-1} - \phi^2$

These four words will represent the addresses of the four estimated phase errors stored in a memory, which errors correspond to these phase change combinations and must be used to calculate the residual errors. Addresses A31, A32, A41 and A42 respectively available at the output of OR circuits 99-102 are inputted to an addressing circuit 103 which controls the addressing of a memory 104. Memory 104 is a 16-position random-access memory (RAM) that permits storing the sixteen possible estimated values of the phase error. Those skilled in the art will appreciate that the four addresses A31, A32, A41 and A42 will be sequentially addressed by circuit 103. The four estimated phase errors read out of memory 104 will be written

$\hat{\theta}_n(A31), \hat{\theta}_n(A32), \hat{\theta}_n(A41)$ and $\hat{\theta}_n(A42)$

These four estimated phase errors are respectively available on lines 105 to 108. For clarity, the output
register of memory 104 has not been shown in the figure, but it will be understood by those skilled in the art that these four errors are successively read out of the memory and stored in the output register to be simultaneously provided on output lines 105 to 108. In order to calculate the four residual errors defined by relations (18) to (21), it is necessary to calculate the phase differences $\psi_n - \phi^1$ and $\psi_n - \phi^2$. To this end, phases $\phi^1$ and $\phi^2$ available in coded form are converted into radians by multiplying same by $\pi/2$. Phase $\phi^1$ present in coded form on line 81 is applied via line 109 to one of the two inputs of a multiplier 110 the other input of which receives the quantity $\pi/2$. Similarly, phase $\phi^2$ present on line 82 is applied via line 111 to one of the two inputs of a multiplier 112 to the other input of which the quantity $\pi/2$ is applied. Note that since phase $\phi^1$ and $\phi^2$ are coded by means of two bits each, the multiplications can be replaced by two additions or by a table look-up operation. The quantity $\psi_n - \phi^1$ is calculated by adder AD21 whose (-) input is connected to the output of multiplier 110 and whose (+) input receives phase $\psi_n$ via line 113. The quantity $\psi_n - \phi^2$ is calculated by adder AD22 whose (-) input is connected to the output of multiplier 112 and to whose (+) input phase $\psi_n$ is applied via line 113. The output of adder AD21 is connected via line 114 to the (+) inputs of adders AD23 and AD24 whose (-) inputs are respectively connected to lines 105 and 107. The output of adder AD22 is connected via line 115 to the (+) inputs of adders AD25 and AD26 whose (-) inputs are respectively connected to lines 106 and 108. The four residual errors noted below are respectively obtained at the
outputs of adders AD23-AD26:

\[ \xi_n(A31) = \psi_n - \phi^1_n - \hat{\phi}_n(A31) \]
\[ \xi_n(A41) = \psi_n - \phi^1_n - \hat{\phi}_n(A41) \]
\[ \xi_n(A32) = \psi_n - \phi^2_n - \hat{\phi}_n(A32) \]
\[ \xi_n(A42) = \psi_n - \phi^2_n - \hat{\phi}_n(A42) \]

Step 2 of the method of the present invention involves comparing the residual errors thus obtained and selecting whichever of phases \( \phi^1 \) and \( \phi^2 \) yields the smallest residual error. These four residual errors are fed via lines 109' - 112' to a selection circuit 113' which selects the address corresponding to the smallest residual error. An exemplary embodiment of circuit 113' is illustrated in figure 4C. The word comprised of the two least significant bits (LSB) of the address of the smallest error represents phase change \( \hat{\phi}_{n-1} - \hat{\phi}_n \). The selection of the two least significant bits takes place in block 114' designated LSB and connected to the output of selection circuit 113'. The phase change \( \hat{\phi}_{n-1} - \hat{\phi}_n \) present on output line 115' of block 114' is compared with phase shifts \( \hat{\phi}_{n-1} - \phi^1 \) and \( \hat{\phi}_{n-1} - \phi^2 \), respectively available on lines 93 and 94, in the comparison and selection circuit 44' which is identical with the comparison and selection circuit 44 of figure 3 and supplies phase \( \hat{\phi}_n \) on its output line 116. The value of \( \hat{\phi}_n \) is

\[ \hat{\phi}_n = \phi^1 \quad \text{if} \quad \hat{\phi}_{n-1} - \hat{\phi}_n = \hat{\phi}_{n-1} - \phi^1 \]
\[ \hat{\phi}_n = \phi^2 \quad \text{if} \quad \hat{\phi}_{n-1} - \hat{\phi}_n = \hat{\phi}_{n-1} - \phi^2 \]
Phase $\phi_n$ is applied via line 117 to the data output line of the detector of the invention, as well as to a delay element 118 which introduces a delay of $T$ seconds and supplies phase $\phi_{n-1}$ on its output line 92.

Adaptive Determination (Step 3)

The adaptive manner in which the estimated phase errors are determined in accordance with expression (22) will now be described.

The addresses A31, A32, A41 and A42 and the corresponding residual errors $\xi_n(A31), \xi_n(A32), \xi_n(A41), \xi_n(A42)$ are respectively fed to delay elements 119-126, each of which introduces a T-second delay. The estimated phase error to be adjusted is that which corresponds to the combination of phase changes $\phi_n-\phi_{n-1}, \phi_{n-2}-\phi_{n-1}$. The address ADR of this estimated phase error is calculated as follows. Phase $\phi_n$ is fed via line 127 to a delay element 128 that introduces a T-sec. delay and supplies phase $\phi_{n-1}$. Phase $\phi_{n-1}$ is subtracted from phase $\phi_n$ by a modulo-4 adder AD27. Phase $\phi_{n-1}$ is also fed to a delay element 129 that introduces a T-sec. delay and supplies phase $\phi_{n-2}$. Phase $\phi_{n-1}$ is subtracted from $\phi_{n-2}$ by a modulo-4 adder AD28. The output of AD27 is multiplied by 4 (two shifts to the left) in multiplier 130. The output of multiplier 130 and that of adder AD28 are concatenated by OR circuit 131 which supplies on its output line 132 the 4-bit address ADR corresponding to the combination of phase changes $\phi_n-\phi_{n-1}, \phi_{n-2}-\phi_{n-1}$. Address ADR is then compared in comparators COMP5-COMP8 with the addresses A31, A32, A41 and A42 associated with the preceding signaling instant and
which are available at the output of delay elements 119-122. To this end, address ADR is applied via line 132 to an input of each of the comparators COMP5-COMP8, the other inputs of which are respectively connected to the outputs of delay elements 119-122 via lines 133-136. The outputs of COMP5-COMP8 are respectively fed to one of the two inputs of AND gates 137-140, the other inputs of which are respectively connected to the outputs of delay elements 123-126. The outputs of AND gates 137-140 are connected to the inputs of an OR gate 141. Thus, one obtains at the output of OR gate 141 the one of the residual errors associated with the preceding signaling instant which corresponds to address ADR. This residual error, which will be written \( \hat{\varepsilon}_{n-1}^{y}(ADR) \), is multiplied by the constant \( y \) in multiplier 142. In the example illustrated in the figure, \( y = 1/32 \) and the multiplication is in fact replaced by a shift of four bits to the right. The quantity \( \mu \hat{\varepsilon}_{n-1}^{y}(ADR) \) is thus obtained at the output of multiplier 142. Address ADR is also applied via line 132 to addressing circuit 103 and causes the corresponding estimated phase error, which will be written \( \hat{\theta}_{n}(ADR) \), to be read out of memory 104. Error \( \hat{\theta}_{n}(ADR) \) available on line 143 is added to \( \mu \hat{\varepsilon}_{n-1}^{y}(ADR) \) in adder AD29 which thus provides the new estimated phase error

\[
\hat{\theta}_{n+1}(ADR) = \hat{\theta}_{n}(ADR) + \mu \hat{\varepsilon}_{n-1}^{y}(ADR)
\]

This new estimated phase error is entered into memory 104 through input register 144 at the address ADR fed via line 132 to addressing circuit 103. The detection of phase \( \hat{\theta}_{n+1} \) can therefore take place at the next signaling instant.
An exemplary embodiment of the address selection circuit 113' of figure 4B will now be described with reference to figure 4C. The residual errors $\xi_n(A31)$ and $\xi_n(A41)$ respectively present on output lines 109' and 110' of adders AD23 and AD24, respectively, are respectively inputted to two circuits 145 and 146 which provide the absolute values $|\xi_n(A31)|$ and $|\xi_n(A41)|$. Absolute value $|\xi_n(A41)|$ is subtracted from absolute value $|\xi_n(A31)|$ in adder AD30, and the sign of the difference thus obtained is detected by sign detector 147. Detector 147 supplies a 0 bit if the difference is negative and a 1 bit if it is positive. The output of detector 147 is directly applied to one of the inputs of each of 2-input AND gates 148 and 149, and is applied through an inverter 150 to one of the inputs of each of 2-input AND gates 151 and 152. The other input of AND gate 149 is connected to the output of circuit 146 while the other input of AND gate 148 receives address A41 from OR gate 101. The other input of AND gate 151 is connected to the output of circuit 145 while the other input of AND gate 152 receives address A31 from OR gate 99. The outputs of AND gates 148 and 152 are connected to the two inputs of an OR gate 153. The outputs of AND gates 149 and 151 are connected to the two inputs of an OR gate 154. The smallest of the absolute values $|\xi_n(A31)|$ and $|\xi_n(A41)|$ is obtained at the output of OR gate 154. Address A31 is provided at the output of OR gate 153 if $|\xi_n(A31)| < |\xi_n(A41)|$; otherwise, address A41 is obtained.

Residual errors $\xi_n(A32)$ and $\xi_n(A42)$ are respectively inputted via lines 111' and 112' to a couple of circuits 155 and 156 which supply the absolute values $|\xi_n(A32)|$ and $|\xi_n(A42)|$, respectively. The outputs of circuits 155 and 156 are respectively connected to the (+) and (−) inputs of an adder AD31 whose
output is connected to a sign detector 157. The output of
sign detector 157 is directly connected to one of the inputs
of each of two-input AND gates 158 and 159, and is connected
through an inverter 160 to one of the inputs of each of two-
input AND gates 161 and 162. The other input of AND gate 159
is connected to the output of circuit 156 while the other
input of AND gate 158 receives address A42 from OR gate 102.
The other input of AND gate 161 is connected to the output
of circuit 155 and the other input of AND gate 162 receives
address A32 from OR gate 100. The outputs of AND gates 158
and 162 are connected to the inputs of an OR gate 163 and
the outputs of AND gates 159 and 161 are connected to the
inputs of an OR gate 164. The smallest of the absolute
values \(|\hat{\varepsilon}_n(A32)|\) and \(|\hat{\varepsilon}_n(A42)|\) is obtained at the output of
OR gate 164. One obtains address A32 at the output of OR
gate 163 if \(|\hat{\varepsilon}_n(A32)| < |\hat{\varepsilon}_n(A42)|\); otherwise, address A42
is obtained. The output of OR gate 164 is subtracted from
that of OR gate 154 in adder AD32 and the sign of the difference
thus obtained is detected by sign detector 166. The output
of detector 166 is directly connected to one of the two
inputs of an AND gate 167, and is connected through an
inverter 168 to one of the two inputs of an AND gate 169.
The other input of AND gate 167 is connected to the output
of OR gate 163, and the other input of AND gate 169 is
connected to the output of OR gate 153. The outputs of AND
gates 167 and 169 are connected to the inputs of an OR gate
170, which supplies the address corresponding to the smallest
residual error.
FIGURE 5 - CASE "C"

Figure 5 illustrates an exemplary embodiment of the apparatus to be added to that illustrated in figure 4 to implement the method of the present invention in Case C, with identification of the two phases of the constellation that are closest to $\psi_n$ and $\psi_{n+1}$. The assumptions and notations previously associated with Case B (apparatus of Figure 4) are equally applicable to Case C.

The phase changes $\phi_{n+1} - \phi_n$, $\phi_{n-1} - \phi_n$, and $\phi_{n-2} - \phi_n$ can assume the following values:

\[
\begin{align*}
\phi_{n+1} - \phi_n &= j \pi/2 & j &= 0, 1, 2, 3 \\
\phi_{n-1} - \phi_n &= k \pi/2 & k &= 0, 1, 2, 3 \\
\phi_{n-2} - \phi_n &= s \pi/2 & s &= 0, 1, 2, 3
\end{align*}
\]

The phase error $\epsilon_n$ which alters received phase $\psi_n$ will be written:

\[
\epsilon_n = \theta_n (\phi_{n+1} - \phi_n, \phi_{n-1} - \phi_n) + x_n (\phi_{n-2} - \phi_n)
\]

As in Case B, the detection of $\phi_n$ takes place at instant $(n+1)t$ since $\psi_n$ and $\psi_{n+1}$ must be known. It is assumed that the estimated values $\hat{\psi}_{n-1}$ and $\hat{\psi}_{n-2}$ are available together with the sixteen estimated values $\hat{\theta}_n$ and the four estimated values $\hat{x}_n$. The method of the present invention, as implemented in the apparatus of figure 4 which incorporates the apparatus of Figure 5, is as follows:
Step 1

To directly perform the previously described step 1 of Case C, it is necessary to calculate the residual errors $e_j$ as follows:

$$e_n^j = r_n - \phi^j - \hat{\theta}_n (\phi^1 - \phi^j, \phi^1 - \phi^j) - \hat{x}_n (\phi^1 - \phi^j)$$

for $j, \ell = 0, 1, 2, 3$.

In practice, this step may be split into two sub-steps as follows:

Step 1-1

This consists in selecting those phases of the constellation, $\phi^1$ and $\phi^2$, which are closest to $\psi_n$, and those phases, $\phi^3$ and $\phi^4$, which are closest to $\psi_{n+1}$.

Step 1-2

This consists in calculating the four residual errors

$$\begin{align*}
\epsilon_n^{31} & = \psi_n - \phi^1 - \hat{\theta}_n (\phi^3 - \phi^1, \phi^3 - \phi^1) - \hat{x}_n (\phi^3 - \phi^1) \\
\epsilon_n^{32} & = \psi_n - \phi^2 - \hat{\theta}_n (\phi^3 - \phi^2, \phi^3 - \phi^2) - \hat{x}_n (\phi^3 - \phi^2) \\
\epsilon_n^{41} & = \psi_n - \phi^1 - \hat{\theta}_n (\phi^4 - \phi^1, \phi^4 - \phi^1) - \hat{x}_n (\phi^4 - \phi^1) \\
\epsilon_n^{42} & = \psi_n - \phi^2 - \hat{\theta}_n (\phi^4 - \phi^2, \phi^4 - \phi^2) - \hat{x}_n (\phi^4 - \phi^2)
\end{align*}$$

Note that

$$\begin{align*}
\epsilon_n^{31} & = \epsilon_n^{31} - \hat{x}_n (\phi^1 - \phi^1) \\
\epsilon_n^{32} & = \epsilon_n^{32} - \hat{x}_n (\phi^2 - \phi^2) \\
\epsilon_n^{41} & = \epsilon_n^{41} - \hat{x}_n (\phi^1 - \phi^1) \\
\epsilon_n^{42} & = \epsilon_n^{42} - \hat{x}_n (\phi^2 - \phi^2)
\end{align*}$$
where $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_4$, and $\varepsilon_4$ are the residual errors calculated in step 1-2 of Case B. Here, these residual errors are partial residual errors only.

**Step 2**

If the smallest residual error is

$$\varepsilon_n^2 \text{ or } \varepsilon_n^4$$

then $\hat{\phi} = \phi^1$

If the smallest residual error is

$$\varepsilon_n^3 \text{ or } \varepsilon_n^4$$

then $\hat{\phi} = \phi^2$

**Step 3**

The two terms $\hat{\theta}_n$ and $\hat{x}_n$ of the phase error are adjusted separately and in an adaptive manner.

The terms $\hat{\theta}_n$ are adjusted as in Case B, except that $e$ will be substituted for $\varepsilon$.

A new estimated value of term $\hat{\theta}_n$ is obtained in accordance with expression

$$(22') \quad \hat{\theta}_{n+1} = \hat{\theta}_n + \hat{\phi}_{n-1} \hat{\phi}_{n-1} + \hat{\phi}_{n-2} \hat{\phi}_{n-2} + \varepsilon_{n-1}$$

where $\varepsilon_{n-1}$ is the residual error observed at the preceding signaling instant and which corresponds to the combination of phase changes

$$\hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-1}$$

The terms $\hat{x}_n$ are adjusted as follows:

Upon completion of step 2, a new estimated value

$$\hat{x}_{n+1} = \hat{\phi}_{n-3} - \hat{\phi}_{n-1}$$

is obtained in accordance with relation (32):
where $\hat{\varepsilon}_{n-1}$ is the residual error found at the preceding signaling instant and which corresponds to the combination of phase changes

$$\hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-1}.$$

The apparatus used to implement the above method in Case C will now be described with reference to figures 4 and 5.

Detection of $\hat{\phi}_n$ (Steps 1 and 2)

Constellation phases $\phi^1$ and $\phi^2$ which are closest to $\psi_n$ and constellation phases $\phi^3$ and $\phi^4$ which are closest to $\psi_{n+1}$ are respectively available on lines 81, 82, 83 and 84 of the apparatus of Figure 4. Addresses A31, A32, A41 and A42 of the terms $\hat{\phi}_n (A31), \hat{\phi}_n (A32), \hat{\phi}_n (A41)$ and $\hat{\phi}_n (A42)$ stored in memory 104 are also available on lines 105-108 of the apparatus of Figure 4. Similarly, $\hat{\varepsilon}_n (A31), \hat{\varepsilon}_n (A32), \hat{\varepsilon}_n (A41)$ and $\hat{\varepsilon}_n (A42)$ remain available at the output of adders AD23, AD24, AD25 and AD26. The calculation of the residual errors defined by relations (28) to (31) makes it necessary to determine the terms $X_n (\hat{\phi}_{n-2} - \phi^1)$ and $X_n (\hat{\phi}_{n-2} - \phi^2)$. Accordingly, phase changes $\hat{\phi}_{n-2} - \phi^1$ and $\hat{\phi}_{n-2} - \phi^2$ must be determined. To this end, phase $\phi^1$ available on line 81 (figure 4) is applied via line 171 to the (-) input of a two-bit (modulo 4) adder AD33 (figure 5) whose (+) input receives via line 172 phase $\hat{\phi}_{n-2}$ available, for example, at the output of delay element 129 of figure 4. Phase $\phi^2$ available on line 82 (figure 4) is applied via line 173 to the (-) input of a two-bit (modulo
4) adder AD34 (figure 5) to whose (+) input phase \( \hat{\phi}_{n-2} \) is applied via line 172. Phase changes \( \hat{\phi}_{n-2} - \phi^1 \) and \( \hat{\phi}_{n-2} - \phi^2 \) respectively available at the output of adders AD33 and AD34 are fed to an addressing circuit 174 that controls the addressing of a memory 175. Memory 175 is a four-position random-access memory (RAM) storing the four possible values of term \( \hat{X}_n \). Addressing circuit 174 controls the sequential read out in memory 175 of terms \( \hat{X}_n (\phi_{n-2} - \phi^1) \) and \( \hat{X}_n (\phi_{n-2} - \phi^2) \) which become simultaneously available on lines 176 and 177.

These terms read out of memory 175 are subtracted from partial residual errors \( \hat{\xi}_n (A31) \), \( \hat{\xi}_n (A41) \), \( \hat{\xi}_n (A32) \) and \( \hat{\xi}_n (A42) \) in accordance with relations (28) to (31). The term \( \hat{X}_n (\phi_{n-2} - \phi^1) \) present on line 176 is applied in parallel to the (-) inputs of a couple of adders AD35 and AD36 to the (+) inputs of which are respectively applied partial residual errors \( \hat{\xi}_n (A31) \) and \( \hat{\xi}_n (A41) \) available at the output of adders AD23 and AD24 of figure 4. The term \( \hat{X}_n (\phi_{n-2} - \phi^2) \) present on line 177 is applied in parallel to the (-) inputs of two adders AD37 and AD38 to whose (+) inputs are respectively applied the partial residual errors \( \hat{\xi}_n (A32) \) and \( \hat{\xi}_n (A42) \) obtained at the output of adders AD25 and AD26 of figure 4. The residual errors \( \hat{\eta}_n (A31) \), \( \hat{\eta}_n (A41) \), \( \hat{\eta}_n (A32) \) and \( \hat{\eta}_n (A42) \) as defined by relations (28) to (31) which thus become available at the output of adders AD35 to AD38 are fed to the selection circuit 113' of figure 4 and also to delay elements 123 to 126, in lieu of errors \( \hat{\xi}_n \) in Case B.

The remainder of the method of detecting \( \hat{\phi}_n \) remains unchanged and phase \( \hat{\phi}_n \) becomes available at the output of OR gate 51' of Figure 4.
Adaptive Determination (Step 3)

The apparatus of Figure 4 provides the term $\hat{\theta}_{n+1}^{(ADR)}$ as in Case B, but this result is achieved by using residual errors $e_{n}(A31)$, $e_{n}(A41)$, $e_{n}(A32)$ and $e_{n}(A42)$ which are now fed to delay elements 123-126, in lieu of residual errors $s_{n}(A31)$, $s_{n}(A41)$, $s_{n}(A32)$ and $s_{n}(A42)$. Relation (22') is written:

$$\hat{\theta}_{n+1}^{(ADR)} = \hat{\theta}_{n}^{(ADR)} + \mu e_{n-1}^{(ADR)}$$

The term $\hat{X}_{n}$ to be adjusted is that which corresponds to phase change $\hat{\phi}_{n-3} - \hat{\phi}_{n-1}$. The address, designated ADR2, of this term in memory 175 is calculated as follows. Phase $\hat{\phi}_{n}$ obtained at the output of OR gate 53' of Figure 4 is inputted to a delay element 178 whose output is connected to the input of another delay element 179. The output of delay element 179 is connected to the input of a delay element 180. The three delay elements 178, 179 and 180 are identical and introduce a T-sec. delay, each. The phase $\hat{\phi}_{n-1}$ obtained at the output of delay element 178 is subtracted in modulo-4 adder AD39 from the phase $\hat{\phi}_{n-3}$ obtained at the output of delay element 180. The phase change $\hat{\phi}_{n-3} - \hat{\phi}_{n-1}$ or address ADR2 is obtained at the output of adder AD39. Address ADR2 is fed to addressing circuit 174 which controls the readout of the term $\hat{X}_{n}(ADR2)$ stored in memory 175. In accordance with relation (32), this term, which is provided on output line 175, is adjusted by using the residual error $e_{n-1}^{(ADR)}$ available at the output of OR gate 141 of Figure 4.

The residual error $e_{n-1}^{(ADR)}$ is multiplied by the constant $\mu$ in multiplier 182 and added in adder AD40 to the term $\hat{X}_{n}(ADR2)$ available on line 181. One thus obtains at the
output of AD40 the term

\[ \hat{X}_{n+1}(ADR2) = \hat{X}_n(ADR2) + \mu \hat{\epsilon}_{n-1}(ADR) \]

which is entered into memory 175 at address ADR2 through input register 183.

The detection of phase \( \hat{\phi}_{n+1} \) can then take place at the next signaling instant.

Those skilled in the art will appreciate that for all their apparent complexity, the devices shown in Figures 3 to 5 will only require a low computing power in practice since they only necessitate additions that mostly involve two-bit words, and comparison operations. Also, most of the components of these devices, memories, addressing circuits, adders, and so forth, would already exist in a receiver using present-day technologies and would not have to be added to implement the inventive method.

From the foregoing description of the method and apparatus of the invention in Cases A, B and C, with identification of those phases of the constellation which are closest to the received phases, it will be understood by those skilled in the art that the method can be used to compensate for the effects of the intersymbol interference created by not more than one leading lobe and/or an arbitrary number of trailing lobes. In its preferred, most general form, the present method, which includes identification of the phases of the constellation, may be defined as follows. (To prevent confusion with Cases A, B and C, as implemented in apparatus illustrated in Figures 3 to 5, some notations have been modified)

Where the interference is created solely by the trailing lobes, the method of the present invention mainly includes the step of:

- selecting those phases, \( \hat{\phi}_1 \) and \( \hat{\phi}_2 \), of the constellation which are closest to received phase \( \psi_n \),
calculating the two residual errors

\[ E_n^L = \psi_n - \phi_n^L - \hat{E}_n^L (\hat{\phi}_{n-1}^L, \hat{\phi}_{n-2}^L, \ldots, \hat{\phi}_{n-N}^L) \]

for \( L = 1, 2 \)

- comparing the residual errors thus obtained,
- selecting as phase \( \hat{\phi}_n \) whichever of phases \( \phi_1^L \) and \( \phi_2^L \) yields the smallest residual error, and
- adjusting the estimated phase error corresponding to \( \hat{\phi}_n \).

Where the intersymbol interference is created by the first leading lobe and \( N \) trailing lobes, the method includes the steps of:

- selecting those constellation phases, \( \phi_1^L \) and \( \phi_2^L \), which are closest to received phase \( \psi_n \), and those constellation phases, \( \phi_3^L \) and \( \phi_4^L \), which are closest to received phase \( \psi_{n+1} \),
- calculating the four residual errors

\[ E_n^{2L} = \psi_n - \phi_n^L - \hat{E}_n^{2L} (\hat{\phi}_n^L, \hat{\phi}_{n-1}^L, \hat{\phi}_{n-2}^L, \ldots, \hat{\phi}_{n-N}^L) \]

for \( L = 1, 2 \) and \( J = 3, 4 \)

- comparing these residual errors with each other,
- selecting as phase \( \hat{\phi}_n \) whichever of phases \( \phi_1^L \) and \( \phi_2^L \) yields the smallest residual error, and
- adjusting the estimated phase error corresponding to the combination of phase changes

\[ \hat{\phi}_n - \hat{\phi}_{n-1}^L, \hat{\phi}_{n-2}^L, \ldots, \hat{\phi}_{n-(N+1)}^L - \hat{\phi}_{n-1}^L \]
in accordance with the residual error which was found at the preceding signaling instant and corresponds to that combination of phase changes.

While the invention has been particularly shown and described with reference to a preferred embodiment thereof, it will be understood by those skilled in the art that numerous changes in form and detail may be made therein without departing from the spirit and scope of the invention.

What is claimed is:
The embodiments of the invention are defined as follows:
The claims defining the invention are as follows:

1. In a data transmission system using the FSK modulation technique wherein the phase of the signal transmitted at each signaling instant may be any one of the phases of a M-phase constellation, a phase detection method supplying an estimated value $\hat{\phi}_n$ of the phase transmitted at signaling instant $nT$ and compensating for the effects of the intersymbol interference created by not more than one leading lobe and/or an arbitrary number $N$ of trailing lobes of the impulse response of the transmission channel, characterized in that it comprises the steps of:

   a) determining the phase $\psi_n$ of the signal received at signaling instant $nT$,

   b) determining at least two residual errors, each of which is obtained by subtracting from received phase $\psi_n$ one of the phases of the constellation and an estimated value of the phase error created by the intersymbol interference and corresponding to that phase of the constellation,

   c) comparing the residual errors thus obtained with each other and selecting the smallest,

   d) selecting as phase $\hat{\phi}_n$ that phase of the constellation which yields the smallest residual error, and

   e) selectively adjusting the estimated phase errors in accordance with the residual errors.
2. Method according to claim 1 supplying an estimated value \( \hat{\phi}_n \) of the phase \( \phi_n \) and compensating for the effects of the intersymbol interference solely created by an arbitrary number \( N \) of trailing lobes of the impulse response of the transmission channel, characterized in that step b) comprises the steps of:

b1) determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to received phase \( \psi_n' \), and

b2) determining the two residual errors

\[
\hat{\psi}_n^L = \phi_n - \phi^L - E_n^L (\hat{\phi}_{n-1}^L, \hat{\phi}_{n-2}^L, \ldots, \hat{\phi}_{n-N}^L),
\]

for \( L = 1 \) and 2

where \( E_n^L \) is the estimated phase error which corresponds to phase \( \phi^L \) and is dependent on the combination of phase changes

\[
\hat{\phi}_{n-1}^L, \hat{\phi}_{n-2}^L, \ldots, \hat{\phi}_{n-N}^L, \text{ and}
\]

\( \hat{\phi}_{n-1}', \hat{\phi}_{n-2}', \ldots, \hat{\phi}_{n-N}' \) are estimated values of previously determined phases

\( \phi_{n-1}', \phi_{n-2}', \ldots, \phi_{n-N}' \)

3. Method according to claim 2, characterized in that step e) is replaced by step e') consisting in:

e') adjusting the estimated phase error corresponding to phase \( \hat{\phi}_n \) in accordance with the smallest residual error.
4. Method according to claim 1 supplying an estimated value \( \hat{\phi}_n \) of phase \( \phi_n \) and compensating for the effects of the intersymbol interference created by the first leading lobe and an arbitrary number \( N \) of trailing lobes of the impulse response of the transmission channel, characterized in that step b) comprises the steps of:

b1) determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to received phase \( \psi_n \), and those phases of the constellation, \( \phi^3 \) and \( \phi^4 \), which are closest to received phase \( \psi_{n+1} \), and

b2) determining the four residual errors

\[
\hat{E}^J_n = \psi_n - \phi^J - \hat{E}^J_n(\phi^J - \phi, \phi_{n-1}^J - \phi, \ldots, \phi_{n-N}^J)
\]

for \( L = 1, 2 \) and \( J = 3, 4 \),

where

\( \hat{E}^J_n \) is the estimated phase error which corresponds to phase change \( \phi^J - \phi \)

and is dependent on the combination of phase changes

\( \phi^J - \phi, \phi_{n-1}^J - \phi, \ldots, \phi_{n-N}^J - \phi \), and

\( \hat{\phi}_{n-1}, \ldots, \hat{\phi}_{n-N} \) are estimated values of previously determined phases \( \phi_{n-1}, \ldots, \phi_{n-N} \).
5. Method according to claim 4, characterized in that step e) is replaced by step e') consisting in:
   e') adjusting the phase error corresponding to the combination of phase changes

   $$\hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-3}, \ldots, \hat{\phi}_{n-(N+1)} - \hat{\phi}_{n-1}$$

   in accordance with the residual error that was observed at the preceding signaling instant and corresponds to the same combination of phase changes.

6. Method according to claim 1 supplying an estimated value $$\hat{\phi}_n$$ of phase $$\phi_n$$ and compensating for the effects of the intersymbol interference created by the first leading lobe and the first two trailing lobes of the impulse response of the transmission channel, characterized in that step b) is replaced by step b') which consists in:
   b') determining at least two residual errors

   $$\psi_{n-\phi^\ell} - \theta_n (\phi_{n} - \phi^\ell, \hat{\phi}_{n-1} - \phi^\ell) - \chi_n (\hat{\phi}_{n-2} - \phi^\ell)$$

   where $$\phi^j$$ and $$\phi^k$$ represent the phases of the constellation $$\hat{\theta}_n (\phi^j - \phi^k, \hat{\phi}_{n-1} - \phi^k)$$ is a first term of the estimated phase error that is created by the first leading lobe and the first trailing lobe and is dependent on the combination of phase changes $$\phi^j - \phi^k, \hat{\phi}_{n-1} - \phi^k, \text{ and}$$
\( \hat{X}_n(\phi_{n-2} - \phi^\ell) \) is a second term of the estimated phase error that is created by the second trailing lobe and is dependent on phase change \( \hat{\phi}_{n-2} - \phi^\ell \), \( \hat{\phi}_{n-1} \) and \( \hat{\phi}_{n-2} \) being estimated values of phases \( \hat{\phi}_{n-1} \) and \( \hat{\phi}_{n-2} \).

7. Method according to claim 6, characterized in that step e) is replaced by step e') consisting in:

   e') selectively and separately adjusting the terms \( \hat{\theta}_n \) and \( \hat{X}_n \) of the estimated phase error in accordance with the residual errors.

8. Method according to claim 6, characterized in that step b') comprises the steps of:

   b'1) determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to received phase \( \psi_n \), and those phases of the constellation, \( \phi^3 \) and \( \phi^4 \), which are closest to received phase \( \psi_{n+1} \), and

   b'2) determining the four residual errors

\[ \hat{\epsilon}_{n,j}^\ell \quad \text{for} \quad \ell = 1, 2 \quad \text{and} \quad j = 3, 4. \]

9. Method according to claim 8, characterized in that step e') comprises the steps of:

   e'1) adjusting the term \( \hat{\theta}_n \) of the estimated phase error that corresponds to the combination of phase

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changes
\[ \hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-1} \]
in accordance with the residual error that was observed at the preceding signaling instant and corresponds to that combination of phase changes, and

\[ e'2) \text{ adjusting the term } \chi_n \text{ of the estimated phase error that corresponds to phase change } \hat{\phi}_{n-3} - \hat{\phi}_{n-1} \text{ in accordance with the residual error used to adjust term } \theta_n \text{ in step e'1).} \]

10. In a data transmission system using the PSK modulation technique wherein the phase of the signal transmitted at each signaling instant may be any one of the phases of a \( M \)-phase constellation, a phase detection apparatus supplying an estimated value \( \hat{\phi}_n \) of the phase \( \phi_n \) transmitted at signaling instant \( nT \) and compensating for the effects of the intersymbol interference solely created by an arbitrary number \( N \) of trailing lobes of the impulse response of the transmission channel, characterized in that it includes:

- input means adapted for receiving the phase \( \phi_n \) at signaling instant \( nT \),
- selection means for determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to received phase \( \phi'_n \),
- storage means for storing the estimated values \( \hat{\phi}_{n-1}, \hat{\phi}_{n-2}, \ldots, \hat{\phi}_{n-N} \) previously determined,
- means for calculating phase changes \( \hat{\phi}_{n-1} - \phi^1, \ldots, \hat{\phi}_{n-N} - \phi^1 \)

and \( \hat{\phi}_{n-1} - \phi^2, \ldots, \hat{\phi}_{n-N} - \phi^2 \).
means for forming a first address representative of the combination of phase changes
\[ \hat{\phi}_{n-1} - \phi^1, \hat{\phi}_{n-2} - \phi^1, \ldots, \hat{\phi}_{n-N} - \phi^1, \]

means for forming a second address representative of the combination of phase changes
\[ \hat{\phi}_{n-1} - \phi^2, \hat{\phi}_{n-2} - \phi^2, \ldots, \hat{\phi}_{n-N} - \phi^2, \]

storage means for storing estimated values of the possible phase errors,

addressing means for controlling the readout of the estimated phase errors \( E^1_n \) and \( E^2_n \) respectively stored at the first and second addresses of memory,

means for calculating phase changes
\[ \psi_n - \phi^1 \text{ and } \varphi_n - \phi^2 \]

means for calculating residual errors
\[ \hat{E}^1_n = \psi_n - \phi^1 - E^1_n \]
\[ \hat{E}^2_n = \psi_n - \phi^2 - E^2_n \]

means for comparing these two residual errors and selecting as phase \( \hat{\phi}_n \) whichever of phases \( \phi^1 \) and \( \phi^2 \) yields the smallest residual error, and

adjustment means for adjusting the estimated phase error that corresponds to \( \hat{\phi}_n \) in accordance with the smallest residual error, and for writing the new estimated phase error thus obtained at the same address.
11. In a data transmission system using the PSK modulation technique wherein the phase of the signal transmitted at each signaling instant can be any one of the phases of a M-phase constellation, a phase detection apparatus supplying an estimated value $\hat{\phi}_n$ of the phase $\phi_n$ transmitted at signaling instant $nT$ and compensating for the effects of the intersymbol interference created by the first leading lobe and an arbitrary number $N$ of trailing lobes of the impulse response of the transmission channel, characterized in that it includes:

- input means adapted for receiving phase $\phi_{n+1}$ at instant $(n+1)T$,
- delay means whose input is connected to said input means and whose output provides phase $\phi_n$ received at instant $nT$,
- said delay means introducing a T-sec. delay,
- first selection means for determining those phases of the constellation, $\phi_1$ and $\phi_2$, which are closest to phase $\phi_n$,
- second selection means for determining those phases of the constellation, $\phi_3$ and $\phi_4$, which are closest to phase $\phi_{n+1}$,
- means for storing the estimated values $\hat{\phi}_{n-1}$, $\hat{\phi}_{n-2}$, ..., $\hat{\phi}_{n-N}$ previously determined,
- means for calculating phase changes $\phi^3 - \phi^1$, $\phi^4 - \phi^1$, $\phi^3 - \phi^2$, $\phi^4 - \phi^2$, $\hat{\phi}_{n-1} - \phi^1$, $\hat{\phi}_{n-2} - \phi^1$, ..., $\hat{\phi}_{n-N} - \phi^2$,
- means for forming a first address representative of the combination of phase changes $\phi^3 - \phi^1$, $\hat{\phi}_{n-1} - \phi^1$, $\hat{\phi}_{n-2} - \phi^1$, ..., $\hat{\phi}_{n-N} - \phi^1$,
- means for forming a second address representative of the combination of phase changes $\phi^3 - \phi^2$, $\hat{\phi}_{n-1} - \phi^2$, $\hat{\phi}_{n-2} - \phi^2$, ..., $\hat{\phi}_{n-N} - \phi^2$,
means for forming a third address representative of the combination of phase changes

\[ \phi^n - \phi^1, \hat{\phi}_{n-1} - \phi^1, \ldots, \hat{\phi}_{n-N} - \phi^1, \]

means for forming a fourth address representative of the combination of phase changes

\[ \phi^n - \phi^2, \hat{\phi}_{n-1} - \phi^2, \ldots, \hat{\phi}_{n-N} - \phi^2, \]

storage means for storing the estimated values of the possible phase errors,

addressing means for controlling the readout of the estimated phase errors

\[ \hat{E}^{31}_n, \hat{E}^{32}_n, \hat{E}^{41}_n \text{ and } \hat{E}^{42}_n \]

respectively stored at the first, second, third and fourth addresses of said storage means,

means for calculating phase changes

\[ \psi^n - \phi^1 \text{ and } \psi^n - \phi^2, \]

means for calculating residual errors

\[ \hat{\psi}^{31}_n = \psi^n - \phi^1 - \hat{E}^{31}_n \]
\[ \hat{\psi}^{32}_n = \psi^n - \phi^2 - \hat{E}^{32}_n \]
\[ \hat{\psi}^{41}_n = \psi^n - \phi^1 - \hat{E}^{41}_n \]
\[ \hat{\psi}^{42}_n = \psi^n - \phi^2 - \hat{E}^{42}_n \]

means for comparing these residual errors and selecting as phase \( \hat{\phi}^n \) whichever of phases \( \phi^1 \) and \( \phi^2 \) corresponds to the smallest residual error, and

adjustment means for adjusting the estimated phase
error corresponding to the address representative of the combination of phase changes

\[ \hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-1}, \ldots, \hat{\phi}_{n-(N+1)} - \hat{\phi}_{n-N} \]

in accordance with the residual error observed at the preceding signaling instant and which corresponds to that combination of phase changes.

12. In a data transmission system using the PSK modulation technique wherein the phase of the signal transmitted at each signaling instant can be any one of the phases of a 4-phase constellation, a phase detection apparatus supplying an estimated value \( \hat{\phi}_n \) of the phase \( \phi_n \) transmitted at signaling instant \( nT \) and compensating for the effects of the intersymbol interference created solely by the first trailing lobe of the impulse response of the transmission channel, characterized in that it includes:

- input means adapted to receive the phase \( \psi_n \) at signaling instant \( nT \),
- selection means for determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to received phase \( \psi_n \),
- means for storing the estimated value \( \hat{\phi}_{n-1} \) previously determined,
- means for calculating phase changes \( \hat{\phi}_{n-1} - \phi^1 \) and \( \hat{\phi}_{n-1} - \phi^2 \),
- means for forming a first address representative of phase change \( \hat{\phi}_{n-1} - \phi^1 \).
means for forming a second address representative of phase change $\hat{\phi}_{n-1} - \phi^2$,

storage means for storing the estimated values of the four possible phase errors,

addressing means for controlling the readout of the estimated phase errors

$$\hat{\epsilon}_n (\hat{\phi}_{n-1} - \phi^1) \text{ and } \hat{\epsilon}_n (\hat{\phi}_{n-1} - \phi^2)$$

respectively stored at the first and second addresses of said storage means,

means for calculating phase changes

$$\phi_n - \phi^1 \text{ and } \phi_n - \phi^2,$$

means for calculating residual errors

$$\xi^1_n = \phi_n - \phi^1 - \hat{\epsilon}_n (\hat{\phi}_{n-1} - \phi^1)$$

$$\xi^2_n = \phi_n - \phi^2 - \hat{\epsilon}_n (\hat{\phi}_{n-1} - \phi^2)$$

means for comparing residual errors $\xi^1_n$ and $\xi^2_n$ and selecting as phase $\phi_n$ whichever of phases $\phi^1$ and $\phi^2$ yields the smallest residual error, and

adjustment means for adjusting the estimated phase error that corresponds to $\phi_n$ in accordance with the smallest residual error, and for writing the new estimated phase error thus obtained at the same address.
13. In a data transmission system using the PSK modulation technique wherein the phase of the signal transmitted at each signaling instant can be any one of the phases of a 4-phase constellation, a phase detection apparatus supplying an estimated value \( \hat{\phi}_n \) of the phase \( \phi_n \) transmitted at signaling instant \( nT \) and compensating for the effects of the intersymbol interference created by the first leading lobe and the first trailing lobe of the impulse response of the transmission channel, characterized in that it includes:

- input means adapted to receive the phase \( \psi_{n+1} \) at signaling instant \( (n+1)t \),
- delay means whose input is connected to said input means and whose output provides phase \( \psi_n \) received at instant \( nT \), said delay means introducing a delay of \( T \) seconds,
- first selection means for determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to phase \( \psi_n \),
- second selection means for determining those phases of the constellation, \( \phi^3 \) and \( \phi^4 \), which are closest to phase \( \psi_{n+1} \),
- means for storing the estimated value \( \hat{\phi}_{n-1} \) previously determined,
- means for calculating phase changes
  \[ \phi^3 - \phi^1, \phi^3 - \phi^2, \phi^4 - \phi^1, \phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^1, \hat{\phi}_{n-1} - \phi^2, \]
- means for forming addresses A31, A32, A41 and A42 respectively representative of the combinations of phase changes
  \[ \phi^3 - \phi^1, \hat{\phi}_{n-1} - \phi^1 \]
  \[ \phi^3 - \phi^2, \hat{\phi}_{n-1} - \phi^2 \]
  \[ \phi^4 - \phi^1, \hat{\phi}_{n-1} - \phi^1 \]
  \[ \phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^2 \]
storage means for storing the estimated values of the possible phase errors,

addressing means for controlling the readout of the estimated phase errors

\[ \hat{\theta}_n (\phi_1, \phi_{n-1} - \phi_1), \hat{\theta}_n (\phi_2, \phi_{n-1} - \phi_2), \hat{\theta}_n (\phi_4, \phi_{n-1} - \phi_4) \]

and \[ \hat{\theta}_n (\phi_4 - \phi_2, \phi_{n-1} - \phi_2) \]

respectively stored at addresses A31, A32, A41 and A42 of said storage means,

means for calculating phase changes

\[ \psi_n - \phi_1 \] and \[ \psi_n - \phi_2 \]

means for calculating residual errors

\[ \varepsilon_{31} = \psi_n - \phi_1 - \hat{\theta}_n (\phi_3 - \phi_1, \phi_{n-1} - \phi_1) \]
\[ \varepsilon_{32} = \psi_n - \phi_2 - \hat{\theta}_n (\phi_3 - \phi_2, \phi_{n-1} - \phi_2) \]
\[ \varepsilon_{41} = \psi_n - \phi_1 - \hat{\theta}_n (\phi_4 - \phi_1, \phi_{n-1} - \phi_1) \]
\[ \varepsilon_{42} = \psi_n - \phi_2 - \hat{\theta}_n (\phi_4 - \phi_2, \phi_{n-1} - \phi_2) \]

means for comparing the residual errors and selecting as phase \[ \hat{\phi}_n \] whichever of phases \[ \phi_1 \] and \[ \phi_2 \] yields the smallest residual error, and

means for adjusting the estimated phase error corresponding to the address representative of the combination of phase changes

\[ \hat{\phi}_n - \hat{\phi}_{n-1}, \hat{\phi}_{n-2} - \hat{\phi}_{n-1} \]

in accordance with the residual error observed at the preceding
14. In a data transmission system using the PSK modulation technique wherein the phase of the signal transmitted at each signaling instant can be any one of the phases of a 4-phase constellation, a phase detection apparatus supplying an estimated value \( \hat{\phi}_n \) of the phase \( \phi_n \) transmitted at signaling instant \( nT \) and compensating for the effects of the intersymbol interference created by the first leading lobe and the first two trailing lobes of the impulse response of the transmission channel, characterized in that it includes:

- input means adapted to receive the phase \( \psi_{n+1} \) at signaling instant \( (n+1)t \),
- delay means whose input is connected to said input means and whose output provides phase \( \psi_n \) received at instant \( nT \), said delay means introducing a delay of \( T \) seconds.
- first selection means for determining those phases of the constellation, \( \phi^1 \) and \( \phi^2 \), which are closest to phase \( \psi_n \),
- second selection means for determining those phases of the constellation, \( \phi^3 \) and \( \phi^4 \), which are closest to phase \( \psi_{n+1} \),
- means for storing the estimated value \( \hat{\phi}_{n-1} \) previously determined,
- means for calculating phase changes
  \[ \phi^3 - \phi^1, \phi^3 - \phi^2, \phi^4 - \phi^1, \phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^1, \hat{\phi}_{n-1} - \phi^2, \]
- means for forming addresses \( A31, A32, A41 \) and \( A42 \) respectively representative of the combinations of phase changes
  \[ \phi^3 - \phi^1, \hat{\phi}_{n-1} - \phi^1 \]
\( \phi^3 - \phi^2, \hat{\phi}_{n-1} - \phi^2 \)

\( \phi^4 - \phi, \hat{\phi}_{n-1} - \phi^1 \)

\( \phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^2 \)

first storage means for storing estimates of the possible values of a first term of phase error that is dependent on the combination of phase changes

\( \phi_{n+1} - \phi_n, \hat{\phi}_{n-1} - \hat{\phi}_n \)

first addressing means for controlling the readout of the estimated values

\[ \hat{\theta}_n (\phi^3 - \phi, \hat{\phi}_{n-1} - \phi^1), \hat{\theta}_n (\phi^3 - \phi^2, \hat{\phi}_{n-1} - \phi^2), \hat{\theta}_n (\phi^4 - \phi, \hat{\phi}_{n-1} - \phi^1) \]

and \( \hat{\theta}_n (\phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^2) \)

respectively stored at addresses A31, A32, A41 and A42 of said first storage means,

means for calculating phase changes

\( \psi_n - \phi^1 \) and \( \psi_n - \phi^2 \)

means for calculating partial residual errors

\[ \nu^{31}_n = \psi_n - \phi^1 - \hat{\theta}_n (\phi^3 - \phi, \hat{\phi}_{n-1} - \phi^1) \]

\[ \nu^{32}_n = \psi_n - \phi^2 - \hat{\theta}_n (\phi^3 - \phi^2, \hat{\phi}_{n-1} - \phi^2) \]

\[ \nu^{41}_n = \psi_n - \phi^1 + \hat{\theta}_n (\phi^4 - \phi, \hat{\phi}_{n-1} - \phi^1) \]

\[ \nu^{42}_n = \psi_n - \phi^2 + \hat{\theta}_n (\phi^4 - \phi^2, \hat{\phi}_{n-1} - \phi^2) \]

means for storing the estimated value \( \hat{\phi}_{n-2} \) previously

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determined,

means for calculating phase changes

\[ \hat{\phi}_{n-2} - \phi^1 \text{ and } \hat{\phi}_{n-2} - \phi^2 \]

means for forming addresses representative of phase changes

\[ \hat{\phi}_{n-2} - \phi^1 \text{ and } \hat{\phi}_{n-2} - \phi^2 \]

second storage means for storing estimates of the possible values of a second term of phase error that is dependent on phase change \( \phi_{n-2} - \phi_n \),

second addressing means for controlling the readout of the estimated values

\[ \hat{\chi}_n (\phi_{n-2} - \phi^1) \text{ and } \hat{\chi}_n (\phi_{n-2} - \phi^2) \]

stored at said addresses representative of phase changes

\[ \hat{\phi}_{n-2} - \phi^1 \text{ and } \hat{\phi}_{n-2} - \phi^2 \]

in said second storage means,

means for calculating residual errors

\[ \begin{align*}
\hat{\omega}_{31}^n & = \omega_{31}^n - \hat{\chi}_n (\hat{\phi}_{n-2} - \phi^1) \\
\hat{\omega}_{32}^n & = \omega_{32}^n - \hat{\chi}_n (\hat{\phi}_{n-2} - \phi^2) \\
\hat{\omega}_{41}^n & = \omega_{41}^n - \hat{\chi}_n (\hat{\phi}_{n-2} - \phi^1) \\
\hat{\omega}_{42}^n & = \omega_{42}^n - \hat{\chi}_n (\hat{\phi}_{n-2} - \phi^2)
\end{align*} \]

means for comparing the residual errors and selecting...
as phase $\hat{\phi}_n$ whichever of phases $\phi^1$ and $\phi^2$ yields the smallest residual error,

means for adjusting the first term of the estimated phase error corresponding to the address representative of the combination of phase changes

$$\hat{\phi}_n - \hat{\phi}_{n-1} - \hat{\phi}_{n-2} - \hat{\phi}_{n-1}$$

in said first storage means, in accordance with the residual error that was obtained at the preceding signaling instant and corresponds to that combination of phase changes, and

means for adjusting the second term of the estimated phase error corresponding to the address representative of the combination of phase changes

$$\hat{\phi}_{n-3} - \hat{\phi}_{n}$$

in said second storage means, in accordance with the residual error that was observed at the preceding signaling instant and corresponds to the combination of phase changes

$$\hat{\phi}_n - \hat{\phi}_{n-1} - \hat{\phi}_{n-2} - \hat{\phi}_{n-1}$$

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FIG. 48

31071M
FIG. 1A
FIG. 1B
FIG. 1C
FIG. 1D

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FIG. 3C