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APPLICATION FOR A STANDARD PATENT

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25 JAN 1983

PATENT OFFICE

HITACHI, LTD., of 5-1, Marunouchi 1-chome, Chiyoda-ku, Tokyo, Japan.

We hereby apply for the grant of a Standard Patent for an invention entitled:

"CONTROL SYSTEM FOR ELECTRIC CARS DRIVEN BY INDUCTION MOTORS"

which is described in the accompanying specification.

Details of basic application(s):

<table>
<thead>
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<th>Number</th>
<th>Convention Country</th>
<th>Date</th>
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<tbody>
<tr>
<td>11551</td>
<td>Japan</td>
<td>29 January, 1982</td>
</tr>
</tbody>
</table>

APPLICATION ACCEPTED AND AMENDMENTS ALLOWED 13-3-87

The address for service is care of DAVIES & COLLISON, Patent Attorneys, of 1 Little Collins Street, Melbourne, in the State of Victoria, Commonwealth of Australia.

Dated this 25 day of January 1983

H. W. Rimington

(a member of the firm of DAVIES & COLLISON for and on behalf of the Applicant).
In support of the Application made for a patent for an invention entitled: "CONTROL SYSTEM FOR ELECTRIC CARS DRIVEN BY INDUCTION MOTORS"

I Akio TAKAHASHI

We of Hitachi, Ltd., of 5-1, Marunouchi 1-chome, Chiyoda-ku, Tokyo, Japan.

do solemnly and sincerely declare as follows:

1. (a) We are the applicant(s) for the patent.

or (b) I am authorized by Hitachi, Ltd.

the applicant(s) for the patent to make this declaration on their behalf.

2. (a) We are the actual inventor(s) of the invention.

or (b) Masahiko IBAMOTO of 434-2, Takeda, Katsuta-shi, Ibaraki-ken, Japan.

Hiroshi NARITA of 885-21, Haratsubo, Ichige, Katsuta-shi, Ibaraki-ken, Japan and

Katsuaki SUZUKI of 1838, Fujigaoka, Sekimoto-cho, Kitaibaraki-shi, Ibaraki-ken, Japan.

are the actual inventor(s) of the invention and the facts upon which the applicant is entitled to make the application are as follows:

the applicant is the assignee of the said actual inventor(s).

3. The basic application as defined by Section 141 of the Act was made in Japan on the 29 January, 1983 by Hitachi, Ltd.

4. The basic application referred to in paragraph 3 of this Declaration was the first application made in a Convention country in respect of the invention the subject of the application.

Declared at Tokyo this 12 day of January 1983.

Akio TAKAHASHI, Patent Attorney
General Manager, Patent Dept.
(Authorized "Signing" Officer)

DAVIES & COLLISON, MELBOURNE and CANBERRA.
Claim

1. A control system for an electric car, comprising: a plurality of induction motors provided to respectively correspond to a plurality of wheels; means provided to respectively correspond to said induction motors for detecting the revolving speeds of the same; a common power transducer for supplying said plural induction motors with electric power; and means for controlling the output frequency of said power transducer on the basis of the outputs of said revolving speed detecting means,

wherein the improvement resides in that said control means includes: means for selecting the revolving speed detecting means which outputs the maximum value signal in said plurality of revolving speed detecting means during a coasting run of said electric car; means for memorizing said revolving speed detecting means selected; and means for controlling the output frequency of said power transducer during a power run of said electric car with reference to the output of said revolving speed detecting means memorized during the power run.
4. A control system for an electric car, comprising: a plurality of induction motors provided to respectively correspond to a plurality of wheels; means provided to respectively correspond to said induction motors for detecting the revolving speeds of the same; a common power transducer for supplying said plural induction motors with an electric power; and means for controlling the output frequency of said power transducer on the basis of the outputs of said revolving speed detecting means,

wherein the improvement resides in that said control means includes: means for selecting the revolving speed detecting means which outputs the maximum value signal in said plurality of revolving speed detecting means during a coasting run of said electric car; means for memorizing said revolving speed detecting means selected; and means for controlling the output frequency of said power transducer during a regeneratively braked run of said electric car with reference to the output of said revolving speed detecting means memorized during the regeneratively braked run.
Complete Specification for the invention entitled:

"CONTROL SYSTEM FOR ELECTRIC CARS DRIVEN BY INDUCTION MOTORS"

The following statement is a full description of this invention, including the best method of performing it known to us: 

- 1 -
The present invention relates to the frequency control of a car which is driven by induction motors, and more particularly to a control system for an induction motor driven car, which is suitable for use for the control of an electric car having a large number of drive axles.

An electric car using induction motors as its traction motors offers the prospect of remarkably high enhancement of the car's adhesion characteristics. It is preferred to provide a control system for the induction motors which is capable of providing constant torque control in order to provide a comfortable ride. Thus, motor current may be held to a constant level, while holding the slip frequency at a constant value.

In order to hold the slip frequency constant, the rotational frequency may be detected by means of a revolving speed detector (such as a pulse generator) coupled to one of the induction motors or wheels so that the frequency derived by adding a constant slip frequency to that rotational frequency (or by subtracting the former from the latter when in a regeneratively braked run) may be used as the operational frequency of a power transducer (e.g., an inverter). Even where the slip frequency is controlled in a variable manner, it is naturally necessary to detect the revolving speed of that induction motor so that the frequency may be controlled on the basis of the output of the induction motor.

Now, in the case of an electric car, motors are coupled to the multiple axles so that the drive power may be dispersed. However, the respective wheels do not always have an equal diameter but may possibly have a diameter of 860 mm, if they are new, or a
diameter of 780 mm if they are worn out. Generally speaking, therefore, the electric motors have irregular revolving speeds.

In this case, a variety of systems have been proposed depending upon which output of which revolving speed detector the control is conducted on the basis of.

More specifically, systems have been proposed including one in which the outputs of all the revolving speed detectors are fed through a low preference circuit so that the minimum speed may always be referred to, another in which the outputs of all the revolving speed detectors are fed through a high preference circuit, on the contrary, so that the maximum speed may be referred to, still another in which the average value of the outputs of all the revolving speed detectors is referred to, and so on. All of the systems thus proposed have their intrinsic advantages and disadvantages, respectively.

It is, therefore, an object of the present invention to improve the traction performance of an electric car which is driven by induction motors.

According to a first aspect of the present invention, there is provided a control system for an electric car, comprising: a plurality of induction motors provided to respectively correspond to a plurality of wheels; means provided to respectively correspond to said induction motors for detecting the revolving speeds of the same; a common power transducer for supplying said plural induction motors with electric power; and means for controlling the output frequency of said power transducer on the basis of the outputs of said revolving speed detecting means,

wherein the improvement resides in that said control means includes: means for selecting the revolving speed detecting means which outputs the
maximum value signal in said plurality of revolving speed detecting means during a coasting run of said electric car; means for memorizing said revolving speed detecting means selected; and means for controlling the output frequency of said power transducer during a power run of said electric car with reference to the output of said revolving speed detecting means memorized during the power run.

According to a second aspect of the present invention, there is provided a control system for an electric car, comprising: a plurality of induction motors provided to respectively correspond to a plurality of wheels; means provided to respectively correspond to said induction motors for detecting the revolving speeds of the same; a common power transducer for supplying said plural induction motors with an electric power; and means for controlling the output frequency of said power transducer on the basis of the outputs of said revolving speed detecting means;

wherein the improvement resides in that said control means includes: means for selecting the revolving speed detecting means which outputs the maximum value signal in said plurality of revolving speed detecting means during a coasting run of said electric car; means for memorizing said revolving speed detecting means selected; and means for controlling the output frequency of said power transducer during a regeneratively braked run of said electric car with reference to the output of said revolving speed detecting means memorized during the regeneratively braked run.
The induction motor corresponding to the aforementioned selected revolving speed detecting means in the first aspect generates the minimum power run torque during the power run so that the corresponding wheels are hard to slip. So far as all the wheels are not slipping therefore, the output of this revolving speed detecting means is proportional to the running speed of the electric car so that the output frequency of the power transducer is not increased even the wheels partially slip. As a result, the constant-speed running characteristics of the induction motors can be effectively utilized so that the adhesion characteristics of the car can be improved as first mentioned.

During the regeneratively braked run of the electric car, the motor corresponding to the selected revolving speed detecting means, mentioned in the second aspect, is generating the minimum braking torque so that the corresponding wheels are hard to skid. So far as all the wheels are not skidding therefore, the output of this revolving speed detecting means is proportional to the running speed of the electric car so that the output frequency does not drop, even if the wheels partially skid. As a result, the constant-speed running characteristics of the induction motors can be effectively utilized so that the adhesion characteristics can be improved.

The invention is further described by way of example only with reference to the accompanying drawings in which:

Figure 1 is a block diagram showing a control
system for an electric car, which is driven by induction motors, in accordance with one preferred embodiment of the present invention;

Figures 2 to 4 are diagrams illustrating the frequency-torque characteristics of the induction motors for explaining the operation of the present invention;

Figure 5 is a block diagram showing another embodiment of the present invention, in which only the frequency control system of Figure 1 is modified;

Figure 6 is a block diagram showing still another embodiment of the present invention, in which only the frequency control system of Figure 1 is modified; and

Figure 7 is a flow chart showing the micro-computer which is shown in the embodiment of Figure 6.

As shown in Figure 1, the electric power collected from an overhead contact line L through a pantograph PA is introduced into a power transducer INV. This power transducer INV is made operative to generate a three-phase alternating current of variable voltage and frequency and is desirably constructed of an inverter, if the line L is supplied with a direct current, and a combination of a converter and an inverter if the line L is supplied with an alternating current. The following description is directed to the case of the inverter. The output 3 \( \phi \) A.C. of the inverter INV is fed to four induction motors IM\(_1\) to IM\(_4\). To these respective motors IM\(_1\) to IM\(_4\), there are respectively connected wheels W\(_1\) to W\(_4\) by which an electric car is
driven.

The inverter INV has its output voltage $V_M$ controlled in the following manner. In a first example, a motor current $I_M$ is detected by means of a current detector CD, and the output voltage $V_M$ of the inverter INV is so controlled that the output of the current detector CD may follow a current instruction $I_{IM}$. As a result, the ratio of the output frequency $f_0$ to the output voltage $V_M$ of the inverter INV takes a constant value. In a second example, the output voltage (i.e., the motor voltage) of the inverter INV is detected by means of a voltage detector VD, the output of which is so controlled as to become proportional to the frequency instructions $f_0$. As a result, the motor current $I_M$ takes a substantially constant value. Therefore, it is arbitrary which control system is to be adopted or whether they are to be mixed or combined, and the construction of a voltage controller DC is determined.

Now, a frequency control system constituting an essential portion of this embodiment of the present invention will be described in the following.

The revolving speeds of the respective induction motors IM$_1$ to IM$_4$ are detected by means of speed detectors such as pulse generators PG$_1$ to PG$_4$. These generators PG$_1$ to PG$_4$ have their outputs expressed in terms of frequency signals $f_1$ to $f_4$. These frequency signals $f_1$ to $f_4$ are introduced into a selective memory circuit SMC. In the shown state which is schematically expressed by means of a selecting switch SS, the one frequency signal $f_1$ is selected. On the other hand, a slip frequency setting unit $S_f$ generates a slip frequency signal $f_s$, and this slip frequency signal $f_s$ and the aforementioned frequency signal $f_1$ are introduced into a frequency calculating circuit $F$, by which
the inverter output frequency signal $f_0$ is calculated. If the electric car is in its power running state, a switch SW is held in the shown state by the action of a power run signal P so that the frequency calculating circuit F executes the calculation of $f_0 = f_1 + f_s$. If the electric car is in its regeneratively braked state, on the contrary, the aforementioned power run signal P disappears. As a result, the switch SW is shifted to the left as shown, so that the calculating circuit F executes the calculation of $f_0 = f_1 - f_s$.

Assuming now that the selective memory circuit SMC be a low preference circuit during the power run of the electric car, as in the prior art, the operations will be described with reference to Figure 2.

During the power run, the output frequencies $f_1$ to $f_4$ of the pulse generators PG$_1$ to PG$_4$ respectively coupled to the induction motors IM$_1$ to IM$_4$ are fed to a low-frequency preference circuit. If the output frequency $f_4$, for example, of those frequencies $f_1$ to $f_4$ is the lowest, the low-frequency preference circuit has the output frequency $f_4'$, to which the slip frequency $f_s$ is added by the action of the frequency calculating circuit F so that the inverter INV operates at the frequency of $f_0 = f_4' + f_s$. The output voltage $V_M$ of the inverter INV is so controlled, as has been described, that the motor current $I_M$ or $V_M/f_0$ may become constant. As a result, the torques respectively generated by the induction motors are dispersed as indicated at $T_1$ to $T_4$ in the frequency-torque characteristics of Figure 2.

It is not assumed that the wheels slip during the power run. Then, the induction motor IM$_4$ generating the highest torque is liable to slip so that
the operating point is shifted from $P_4$ along as indicated by the arrowed thick solid line in Figure 2. As a result, the output frequency of the low-frequency preference circuit is shifted to $f_3$. Thus, the 5 frequency calculating circuit $F$ executes the calculation of $f'_0 = f_3 + f_s$ so that the operating frequency of the inverter is slightly increased from $f_0$ to $f'_0$. However, this increase is not so large as that of the speed of the slipped wheels. This results in a drop in the probability that the increase of the speeds of the slipped wheels is positively fed back to invite highly idle revolutions.

On the other hand, in case slip take place in the system using the high-frequency preference circuit, the reference frequency may be shifted to the pulse generators of the wheels which have slipped. This leads to large slipping.

If the low-frequency preference circuit is used, as has been described hereinbefore, it is expected that the adhesion characteristics are enhanced to an excellent level. However, the frequency-torque characteristics of all the induction motors after onset of slipping are illustrated by the broken line $T'$ of Figure 2. If the induction motor $IM_3$ is taken up as an example, therefore, the operating point is shifted from $P_3$ to $P'_3$, and the generated torque is increased from $T_3$ to $T'_4$. At this time, the wheel $W_3$ is that which is the most liable to slip. If the wheel $W_3$ slips too, the speed reference is shifted to the wheel $W_2$ while leaving a high possibility that large slippings are reached in a chain reaction.

Thus, even if the slip take place, the speed reference is instantly shifted to the remaining wheels which do not slip so that the frequency is not increased to a high level. However, because the
speed of the wheels generating the maximum torque (so that they are the most liable to slip) is referred to, the speed reference is so variable as to make the frequency control unstable.

On the contrary, the selective memory circuit SMC used in one embodiment of the present invention is constructed in the following manner. A calculating unit CL receives the frequency signals \( f_1 \) to \( f_4 \) during the coasting run of the electric car and selectively memorizes the frequency signal indicating the maximum and the frequency signal indicating the minimum. If the signal \( f_1 \) indicates the maximum and the signal \( f_4 \) indicates the minimum, more specifically, not their values but the frequency \( f_1 \) or \( f_4 \) itself is memorized.

During the power run, moreover, the frequency signal \( f_1 \) at that time is used as the frequency reference. During the regeneratively braked run, on the contrary, the frequency signal \( f_4 \) at that time is used as the frequency reference. The selecting switch SS is changed over in a manner to ensure the selective use of the frequency signals \( f_1 \) and \( f_4 \). On the other hand, the calculating unit CL generates an adjusting signal AD, if necessary, from the relationship among the frequency signals \( f_1 \) to \( f_4 \) during the coasting run.

Now, during the coasting run of the electric car in which the wheels neither slip nor skid, it is assumed that inequalities of \( f_1 > f_2 > f_3 > f_4 \) hold among the output frequencies \( f_1 \) to \( f_4 \) of the respective pulse generators \( PG_1 \) to \( PG_4 \). The selective memory circuit SMC compares the output frequencies \( f_1 \) to \( f_4 \) of the respective pulse generators \( PG_1 \) to \( PG_4 \) during the coasting run and judges that the frequency \( f_1 \) is the highest whereas the frequency \( f_4 \) is the lowest. The selective memory circuit SMC memorizes that the selecting switch SS should be
switched to the pulse generator PG\(_1\) during the power run and to the pulse generator PG\(_4\) during the regeneratively braked run.

With the construction thus far described, the inverter INV is run (as referred to Figure 3(A)) by using the frequency \((f_1 + f_s)\), which is derived by adding the slip frequency \(f_s\) to the highest reference frequency detected, as its operating frequency \(f_0\), as illustrated in Figure 3, so that the respective induction motors IM\(_1\) to IM\(_4\) generate the torques \(T_1\) to \(T_4\).

Here, if idle revolutions take place, the wheel \(W_4\) generating the highest torque slip so that the induction motor IM\(_4\) has its operating point shifted, as indicated at an arrow of a thick line in Figure 3. At this time, the frequency reference is not influenced in the least even if the wheel \(W_4\) idly revolves, because it still resorts to the wheel \(W_1\), so that the operational frequency of \(f_0 = f_1 + f_s\) of the inverter is not abruptly increased. As a result, the torque of the slipped wheel is promptly dropped along the frequency-torque characteristic curve of Figure 3 so that the adhesion is liable to be restored.

During the regeneratively braked run, on the other hand, the frequency \((f_4 - f_s)\), which is derived by subtracting the slip frequency \(f_x\) from the lowest reference frequency \(f_4\) detected in accordance with the frequency-torque characteristic curve illustrated in Figure 4, is used as the operational frequency of the inverter INV (as referred to Fig. 4(A)) so that the respective induction motors generate the torques \(T_1\) to \(T_4\).

If the skid run should take place, the wheel \(W_1\) having the highest braking torque would skid so that only the induction motor IM\(_1\) would have its operating
point shifted as indicated by the arrow in the same Figure. However, since the frequency reference is still provided by the wheel $W_4$ during the regeneratively braked run, no influence is exerted even if the wheel $W_1$ skids, so that the operational frequency $f_0$ of the inverter will not be abruptly dropped. As a result, the braking torque of the wheel having skidded is promptly dropped so that the adhesion is liable to be restored.

Here, since the wheel diameters are dispersed during both the power run and the regeneratively braked run, the torques respectively generated by the motors are dispersed among $T_1$ to $T_4$, as illustrated in Figures 3 and 4. On the other hand, the slip frequency $f_s$ should be set by the necessary total torque ($T_1 + T_2 + T_3 + T_4$) of the electric car. If the wheel diameters are dispersed, therefore, the slip frequency should be adjusted in a manner to take the dispersion into consideration.

On the other hand, it is also possible to automatically make corrections from the relationship among the levels of the frequency signals $f_1$ to $f_4$ during the coasting run. More specifically, the adjusting signal AD shown in Figure 1 corrects the slip frequency $f_s$ from the relationship of the frequency signal $f_1$ or $f_4$ selected with all the frequency signals $f_1$ to $f_4$, e.g., their average value. By this correction, the power run or regeneratively braked run characteristics of the electric car can be maintained at the preset desirable ones.

Figure 5 shows the construction of only that frequency control system of another embodiment of the present invention, which is modified from that of Figure 1.

This embodiment is characterized in that a multi-
plier MU is disposed in the selective memory circuit SMC without correcting a slip frequency $f'_s$ relative to the dispersion among the wheel diameters so that the frequency signal (e.g., $f_1$) selected may be multiplied by a constant $K$ which is determined by the calculating circuit CL.

This results in the calculation of $f_0 = Kf_1 + f'_s$ or $f_0 = Kf_4 - f'_s$, as illustrated in (B) of Figures 3 and 4. As a result, an effect like that of the foregoing embodiment can be obtained.

In the embodiments of Figures 1 and 5, the selective memory circuit SMC is exemplified by the selecting switch SS, which can naturally be replaced by stationary switching means such as a semiconductor.

Figure 6 shows still another embodiment of the frequency control system according to the present invention, in which the selective memory circuit SMC and the frequency calculating circuit F are replaced by a microcomputer MC.

Figure 7 is a chart showing the flow which is to be processed in accordance with the program of the microcomputer MC of Figure 6.

In accordance with the position of the handle of a master controller or in response to the stop signal of the inverter, there is introduced a signal indicating that the car driven by the induction motors is in its coasting run. During this coasting run, the highest frequency is selected, upon which it is determined that the pulse generator generating that signal should be the one to be selected during the power run. Moreover, the minimum frequency is selected, upon which it is determined that the pulse generator generating that signal is the one to be selected during the regeneratively braked run.

During the running operation, on the other hand,
it is judged whether the run is of the power type or the regeneratively braked type. The output frequency of the pulse generator, which has been determined to be selected during the coasting run, is designated at $g_n$, and the calculation of $f_0 = f_n + f_s$ is executed during the power run whereas the calculation of $f_0 = f_n - f_s$ is executed during the regeneratively braked run, thereby to determine the operational frequency of the inverter. By repeatedly executing a series of these processings at a predetermined high-speed rate, it is possible to exhibit the function like that of the embodiment shown in Figure 1.

Thus, if the judgements of the conditions and the processings of the signals are conducted in accordance with the software of the microcomputer, the parts such as the selecting switch can be dispensed with so that the circuit construction can be simplified with an increase in the reliability. This effect is prominent in case the control system is equipped with the microcomputer for another application.

As has been described hereinbefore, according to the present invention, even if slip is occurred, the operational frequency of the inverter is not abruptly fluctuated so that the adhesion can be easily restored with an improvement.
THE CLAIMS DEFINING THE INVENTION ARE AS FOLLOWS:-

1. A control system for an electric car, comprising: a plurality of induction motors provided to respectively correspond to a plurality of wheels; means provided to respectively correspond to said induction motors for detecting the revolving speeds of the same; a common power transducer for supplying said plural induction motors with electric power; and means for controlling the output frequency of said power transducer on the basis of the outputs of said revolving speed detecting means,

wherein the improvement resides in that said control means includes: means for selecting the revolving speed detecting means which outputs the maximum value signal in said plurality of revolving speed detecting means during a coasting run of said electric car; means for memorizing said revolving speed detecting means selected; and means for controlling the output frequency of said power transducer during a power run of said electric car with reference to the output of said revolving speed detecting means memorized during the power run.

2. A control system according to Claim 1, wherein said frequency control means has means for determining the output frequency of said power transducer by adding the output of slip frequency setting means to the output of said revolving speed detecting means memorized during the power run.

3. A control system according to Claim 1, wherein said frequency control means has: means for correcting the output of said revolving speed detecting means memorized during the power run; and means for determining the output frequency of said power transducer
by adding the output of said slip frequency setting means to the output of said correcting means.

4. A control system for an electric car, comprising: a plurality of induction motors provided to respectively correspond to a plurality of wheels; means provided to respectively correspond to said induction motors for detecting the revolving speeds of the same; a common power transducer for supplying said plural induction motors with an electric power; and means for controlling the output frequency of said power transducer on the basis of the outputs of said revolving speed detecting means, wherein the improvement resides in that said control means includes: means for selecting the revolving speed detecting means which outputs the maximum value signal in said plurality of revolving speed detecting means during a coasting run of said electric car; means for memorizing said revolving speed detecting means selected; and means for controlling the output frequency of said power transducer during a regeneratively braked run of said electric car with reference to the output of said revolving speed detecting means memorized during the regeneratively braked run.

5. A control system according to Claim 4, wherein said frequency control means has means for determining the output frequency of said power transducer by subtracting the output of slip frequency setting means from the output of said revolving speed detecting means memorizing during the braked run.

6. A control system according to Claim 4, wherein said frequency control means has: means for correcting the output of said revolving speed detecting means
memorized during the braked run; and means for determining, the output frequency of said power transducer by subtracting the output of said slip frequency setting means from the output of said correcting means.

7. A control system for an electric car, substantially as hereinbefore described with reference to the accompanying drawings.

8. The steps or features disclosed herein or any combination thereof.

DATED this 25th day of January, 1983
HITACHI, LTD.
by its Patent Attorneys
DAVIES & COLLISON.
FIG. 2

TORQUE

T4
T3
T2
T1

0

f1
f2
f3
f4
f0
f0'

FREQUENCY

P1
P2
P3
P4

T

T'
FIG. 3
FIG. 7

START

READ $f_1 - f_4$

COASTING?

NO

YES

MAX. OF $f_1 - f_4$

POWERING

MIN. OF $f_1 - f_4$

POWERING

DETERMINE AND MEMORIZE OF NO. OF P.G. WHICH SHOULD BE USED IN POWERING

DETERMINE AND MEMORIZE OF NO. OF P.G. WHICH SHOULD BE USED IN BRAKING

RETURN

$\begin{align*}
&f_n = \text{FREQUENCY OF P.G. WHICH SHOULD BE USED IN POWERING} \\
&f_0 = f_n + f_s \\
&f_n = \text{FREQUENCY OF P.G. WHICH SHOULD BE USED IN BRAKING} \\
&f_0 = f_n + f_s
\end{align*}$